2-5596 Mechanika viazaných mechanických systémov (VMS)
pre špecializáciu Aplikovaná mechanika, 4.roč. zimný sem.
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2. Vektorová metóda kinematickej analýzy VMS

- Analýza polohy členov VMS,
- Newton-Raphsonova iteračná metóda.
- Konvergenčné predpoklady.
- Prínosy modifikovanej Newton-Raphsonovej iteračnej metódy.
- Analýza rýchlostí a zrýchlení bodov a členov VMS.

Kinematická analýza — Cieľom kinematickej analýzy daného rovinného mechanizmu pre predpísaný pohyb vstupného hnacieho člena/členov je určiť časový priebeh počtu $d = 2k + s_1$ závislých súradníc polohy výstupných členov, kde k je počet základných slučiek a s_1 je počet spojení triedy $t = 1$.

Poskladanie mechanizmu — Prvým krokom pred numerickou kinematickou analýzou daného rovinného mechanizmu so známymi začiatočnými hodnotami závislých súradníc polohy členov a konštantnou hodnotou súradnice polohy vstupného hnacieho člena je poskladanie mechanizmu vo východiskovej konfigurácii s predpísanou toleranciou v podmienke uzatvorenosti základnej slučky členov.

Slučkové rovnice — Na určenie časového priebehu počtu d závislých súradníc polohy výstupných členov je potrebné zostaviť počet d lineárne nezávislých algebrických rovníc podľa typu mechanizmu:

- V rovinnom uzatvorenom mechanizme s rotačnými R a posuvnými P spojeniami je počet $s_1 = 0$, potom počet $d = 2k$ priebehov neznámych závislých súradníc polohy výstupných členov určíme z počtu $d = 2k$ explicitných skalárnych slučkových rovníc, ktoré získame priemetom vektorových slučkových rovníc do smerov osí x_1, y_1

  $$k_j \approx \sum_{i=1}^{p_h} \overline{h}_i = \overline{0},\ j = 1,2,...,k,\ i = 1,2,...,p_h$$

  kde p_h je počet orientovaných strán mnohouhlolníka získaného z kinematickej schémy daného rovinného mechanizmu.

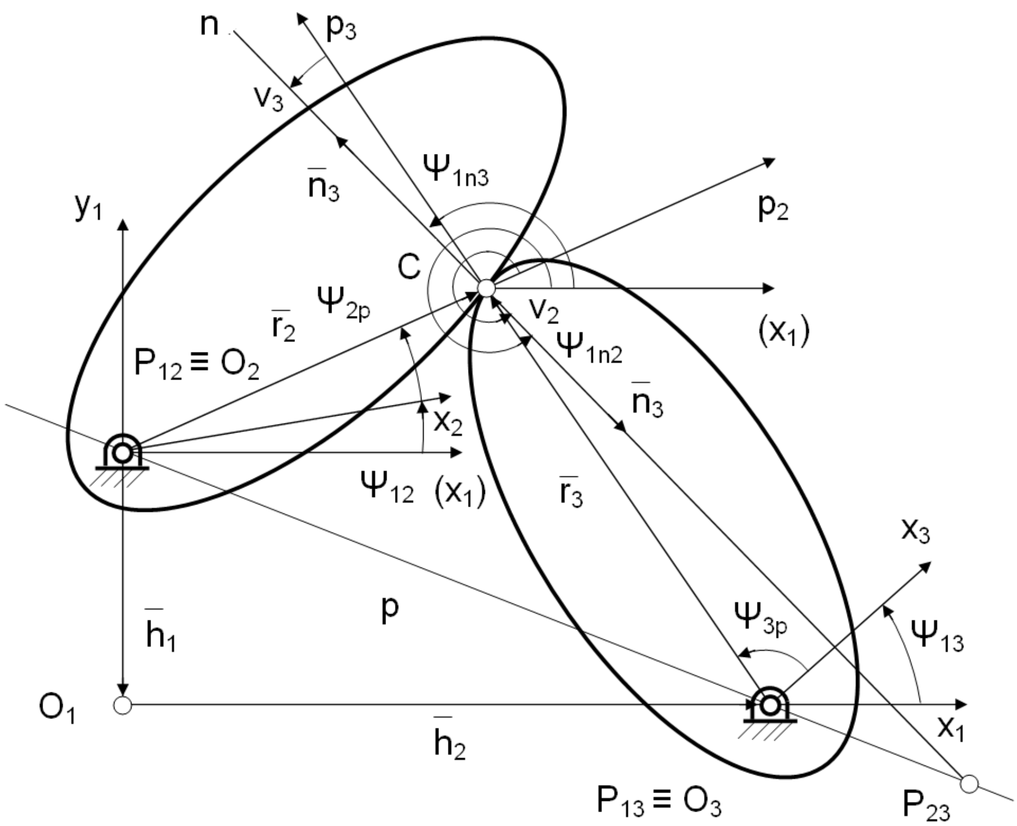
- V rovinnom uzatvorenom vačkovom mechanizme s korektným preklzajúcim K spojením (geometrickou väzbou) typu $t = 1$, v ktorom sa vzájomne dotýkajú hladké povrchy prvkov členov $j = 2$ and $j + 1 = 3$, je potrebné doplniť počtet $d = 2k$ explicitných skalárnych slučkových rovníc o prídavnú explicitnú skalárnu slučkovú rovnicu

  $\psi_{1n(j+1)} - \psi_{1nj} = b\pi$, kde

$\psi_{1nj} = \psi_{1j} + \psi_{jp} + \nu_j$, a $\psi_{1j} = \sphericalangle(x_1, x_j)$ sú globálne súradnice polohy členov a uhol $\psi_{jp} = \sphericalangle(x_j, p_j)$, kde p_j sú nositeľky daných polohových vektorov $\vec{r}_j = F_j(\vec{\psi}_{jp})$, ktoré definujú tvar stýkajúcich sa povrchov a uhly $\vec{\nu}_j = \sphericalangle(\vec{r}_j, \vec{n}_j)$ zvierajú polohové vektory $\vec{r}_j$ s vonkajšími normálovými vektormi $\vec{n}_j$.

Uhly $\vec{\nu}_j$ vyplývajú z daného tvaru kontaktných kriviek podľa dvojargumentovej funkcie $\vec{\nu}_j = \mathrm{arctg}_2(y/x)$, so znamienkom priemetov y a x v kvadrantoch jednotkovej kružnice.

Súčiniteľ b vyplýva z východiskovej polohy dotýkajúcich sa povrchov.



Obr.1 Preklzajúce K spojenie vo vačkovom mechanizme.

- In a planar closed mechanism the closed rolling joint V is of class $t = 2$, but singular configuration of adjacent links j and $j+1$ with mating circular shapes causes that rolling joint is incorrect, partially passive, with number $n_N = 1$ of uneliminated degrees of freedom. So actual mobility n_S of mechanism with number r_V of closed rolling joints is then
  $n_S = n + r_V\, n_N$
  where n is mobility of mechanism evaluated under assumption of its correctness.

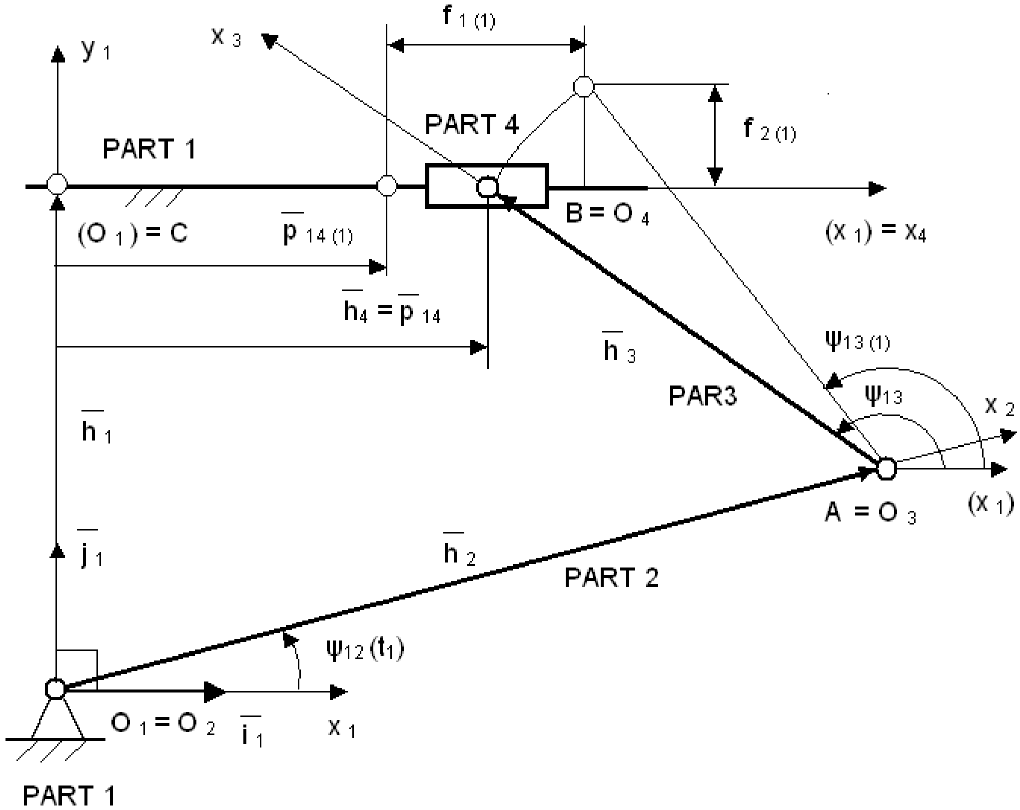
- In a planar closed mechanism with open rolling joint V each open rolling joint V should be transformed into closed rolling joint imposing the auxilliary fictive binary link into mechanism. Each basic loop in mechanism with closed rolling joints of links with circular shapes degenerates into abscissa.
  Because the number $d = 2k + s_1$ of dependet global position coordinates have to be determined, it is necessary add to the number $d = 2k$ of explicit scalar loop constraint equations one auxilliary explicit constraint equation of pure rolling condition resulting from basic equation of planetary (epicyclic) gear train
  $R_C \omega_{1C} = (R_P + R_C)\omega_{1R} - R_P \omega_{1P}$
  where
  R_C is diameter of sun gear with angular velocity ω_{1C},
  R_R is length of arm (spider, or carrier) rotating with angular velocity ω_{1R},
  R_P is diameter of planet gear with angular velocity ω_{1P}.

Príklad

Úlohou kinematickej analýzy kľukového mechanizmu podávača z obr. 2 je určiť časový priebeh závislých globálnych súradníc $\psi_{z\,i}$, $i = 1, 2, ..., d$ polohy členov
$\psi_{z1} = \psi_{13}$, $\psi_{z2} = \overline{p}_{14}$

Celkový počet m globálnych súradníc
ψ_i, $i = 1, 2, ..., m$, kde
$m = n + d$,
n je počet nezávislých globálnych súradníc polohy hnacích členov (n je zároveň pohyblivosť mechanismu)
$\psi_{n\,i}$, $i = 1, 2, ..., n$
d je počet závislých globálnych súradníc polohy členov
$\psi_{z\,i}$, $i = 1, 2, ..., d$.

The total number m of global relative position coordinates ψ_i, $i = 1,2,...,m$ of the links is a sum $m = n + d$ where n is number of independent global position coordinates of input links $\psi_{n\,i}$, $i = 1,2,...,n$, ($\psi_{n1} = \psi_{13}$ in our example) while n is mobility of mechanism and d is number of dependent global position coordinates of the links $\psi_{z\,i}$, $i = 1,2,...,d$, ($\psi_{z1} = \psi_{13}$, $\psi_{z2} = \overline{p}_{14}$ in our example).



Obr.2 Východisková konfigurácia členov kľukového mechanizmu podávača v čase $t = t_1$.

Slučkové rovnice

In our sigle loop slider crank mechanism from Fig.2 the closure condition of polygon is

$$\sum_{i=1}^{p_h} \overline{h}_i = \overline{0}\,,\ i = 1,2,...,p_h \tag{1}$$

where p_h is the number of oriented edges $\overline{h}_i$ in the polygon related to given planar mechanism and $\overline{h}_4 = \overline{p}_{14}$. Then vectorial loop equation according to direction of adding is

$$-\overline{h}_1 + \overline{h}_2 + \overline{h}_3 - \overline{p}_{14} = \overline{0} \tag{2}$$

After multiplying Eq. (2) by unit vectors $\overline{i}_1, \overline{j}_1$ we obtain scalar constraint equations as the projections of vectors into axes x_1, y_1

$$-h_1 c\alpha_1 + h_2 c\psi_{12} + h_3 c\psi_{13} - p_{14} c\alpha_4 = 0 \tag{3}$$
$$-h_1 s\alpha_1 + h_2 s\psi_{12} + h_3 s\psi_{13} - p_{14} s\alpha_4 = 0 \tag{4}$$

where $\alpha_1 = \sphericalangle (x_1, y_1)$ and $\alpha_4 = \sphericalangle (x_1, x_4)$

Väzobné rovnice

Using arithmetic vector notation we can write scalar constraint equations (3), (4) in the form

$$\mathrm{f_i}(\bar{\psi}_z) \approx \mathrm{f_i}(\psi_{z1},...,\psi_{zd}) = 0,\ \mathrm{i} = 1,2,...,\mathrm{d} \tag{5}$$

It is always possible to determine the time course for number d of unknown dependent global position coordinates ψ_{zi}, $i = 1,2,...,d$ of output links in the arithmetic vector $\bar{\psi}_z = \lceil \psi_{z1},\ldots,\psi_{zd} \rceil$ for known arithmetic vector $\bar{\psi}_n = \lceil \psi_{n1},\ldots,\psi_{nn} \rceil$ of independent global position coordinates of the input links, where n is mobility of mechanism, solving nonlinear algebraic constraint equations (5) by numerical Newton-Raphson (N-R) iteration method.

Assembly process

The very first step in the kinematic analysis of given mechanism is the assembly process with prescribed tolerance for closure condition in the initial configuration of mechanism (see Fig.1) when input independent global position coordinates at the time $t = t_1$ are constant.

Newton-Raphson

The goal of the N-R method is to find the root $\bar{\psi}_z$ of a function (5), that is to find new $\bar{\psi}_{z(2)}$ such that $\bar{f}(\bar{\psi}_{z(2)}) = \bar{0}$ if one knows the value of previous $\bar{\psi}_{z(1)}$ for $\bar{f}(\bar{\psi}_{z(1)}) = \bar{0}$ and the value of all the partial derivatives $\dfrac{\partial \bar{f}(\bar{\psi}_{z(1)})}{\partial \bar{\psi}_z}$, $\dfrac{\partial^2 \bar{f}(\bar{\psi}_{z(1)})}{\partial \bar{\psi}_z^{\,2}}$, etc. Rather than computing all the derivatives, the series is truncated after the first derivative. The assembly process is repeated until the difference between two successive approximations is less than a small number ε representing prescribed tolerance.

Linearizácia

In the initial configuration of a mechanism the nonlinear algebraic constraint equations $\bar{f} = \lceil \mathrm{f}_1,...,\mathrm{f}_d \rceil$ can be linearized by linear terms of Taylor series approximation in a summation with residuals $\bar{f}_{(r)}$

$$\bar{f} \cong \bar{f}_{(r)} + V_{(r)} \Delta \bar{\psi}_{z\,(r)} \tag{6}$$

where r is the number of iteration step,

Reziduá

residuals $\bar{f}_{(r)}$ are obtained after introduction of the arithmetic vector $\bar{\psi}_{z\,(r)}$ of estimated position coordinates into constraint equations (5), matrix $V_{(r)}$ is Jacobi matrix of the rank $(d \times d)$

Jakobián

$$V_{(r)} = \left[\frac{\partial \mathrm{f_i}}{\partial \psi_{zj}} \right]_{(r)},\ \mathrm{i} = 1,2,...,\mathrm{m},\ \text{and}\ \mathrm{j} = 1,2,...,\mathrm{d} \tag{7}$$

Korekcie

and $\Delta\bar{\psi}_{z\,(r)}$ is unknown arithmetic vector of corrections. The arithmetic vector of nonlinear algebraic constraint equations is $\bar{f} = \bar{0}$ after introducing the exact solution $\bar{\psi}_z$. It stands to reason that residuals $\bar{f}_{(r)}$ are nonzero $\bar{f}_{(r)} \neq \bar{0}$ after introducing the arithmetic vector $\bar{\psi}_{z\,(r)}$ of estimated dependent global position coordinates into constraint equations (5), then also $\bar{f} \neq \bar{0}$.

Numerické riešenie

The arithmetic vector $\bar{\psi}_{z\,(r+1)}$ will be the accepted solution, if the norm $\left\|\bar{f}_{(r)}\right\|$ of residuals satisfy condition $\left\|\bar{f}_{(r)}\right\| \leq \varepsilon$, so maximum residual from all residuals is less than a specified tolerance ε. This can be achieved by iteration process of converged (N-R) method

$$\bar{\psi}_{z\,(r+1)} = \bar{\psi}_{z\,(r)} + \Delta\bar{\psi}_{z\,(r)},\ r = 1,2,\ldots,p \tag{8}$$

which will be finished, if the norm $\left\|\Delta\bar{\psi}_{z\,(r)}\right\|$ of corrections satisfy the condition

$$\left\|\Delta\bar{\psi}_{z\,(r)}\right\| \leq \varepsilon \tag{9}$$

where ε is prescribed tolerance. Then set $\bar{\psi}_{z\,(r+1)}$ of dependent global position coordinates satisfies all constraint equations (5).

In our example on Fig.1 the constraint equations (3),(4) are linearized according to Eq.(6)

$$f_1 \cong f_{1(1)} + \left[\frac{\partial f_1}{\partial \psi_{13}}\right]_{(1)} \Delta\psi_{13(1)} + \left[\frac{\partial f_1}{\partial p_{14}}\right]_{(1)} \Delta p_{14(1)} \tag{10}$$

$$f_2 \cong f_{2(1)} + \left[\frac{\partial f_2}{\partial \psi_{13}}\right]_{(1)} \Delta\psi_{13(1)} + \left[\frac{\partial f_2}{\partial p_{14}}\right]_{(1)} \Delta p_{14(1)} \tag{11}$$

For updation of dependent global position coordinates $\psi_{13(2)}$, $p_{14(2)}$ in second iteration step and improvement of initial estimation $\psi_{13(1)}$, $p_{14(1)}$, there are used corrections $\Delta\psi_{13(1)}$, $\Delta p_{14(1)}$ obtained solving linear equations (10), (11)

$$\psi_{13(2)} = \psi_{13(1)} + \Delta\psi_{13(1)} \tag{12}$$

$$p_{14(2)} = p_{14(1)} + \Delta p_{14(1)} \tag{13}$$

Kinematická analýza

The goal of the position-level kinematic analysis is to find $\bar{\psi}_z(t_2)$ for increment $\text{in}\,\bar{\psi}_n(t_1) = \bar{\psi}_n(t_2) - \bar{\psi}_n(t_1)$ of input independent position coordinates (see Fig.2) corresponding to the defined time step $h = t_2 - t_1$. The vector $\bar{\psi}_z(t_1)$ obtained from assembly process at the time $t = t_1$ is now the estimate for N-R method.

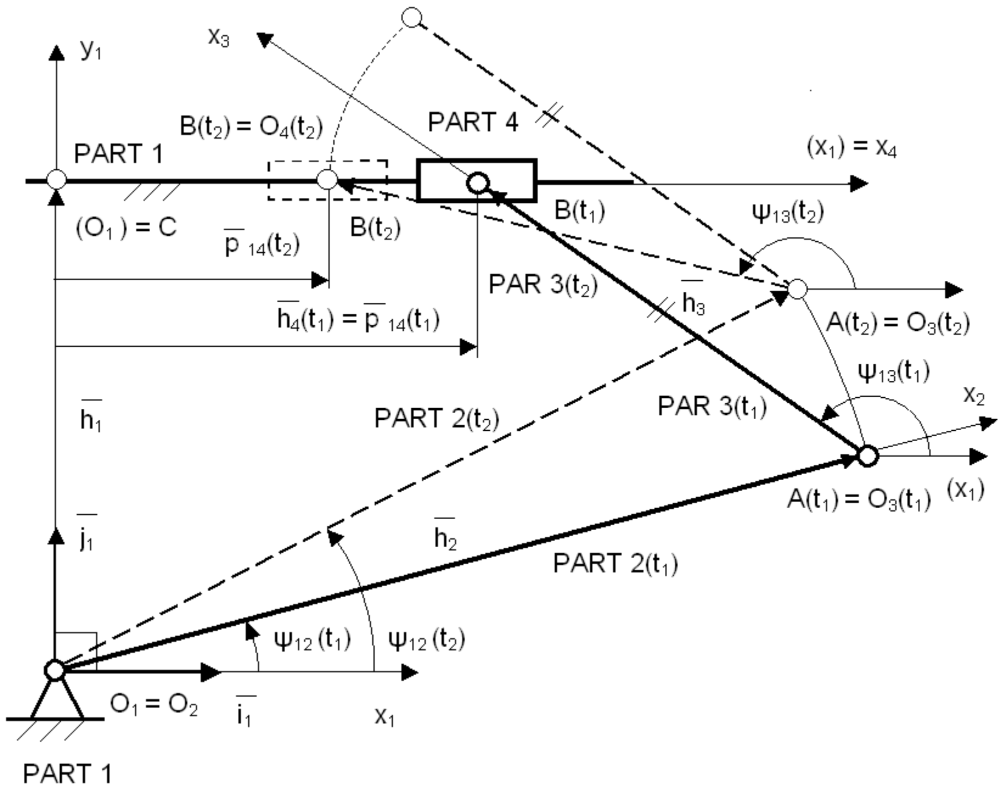


Fig.2 Konfigurácie členov kľukového mechanizmu podávača v časových krokoch t_1, t_2.

Analýza polohy

Goal of the position kinematic analysis of the given planar mechanism with known arithmetic vector $\bar{\psi}_n = \lceil \psi_{n1},...,\psi_{nn} \rceil$ of independet global position coordinates of the input link/or links is to determine the time course for number $d = 2k + s_1$ of unknown dependet global position coordinates of ouput links in the arithmetic vector $\bar{\psi}_z = \lceil \psi_{z1},...,\psi_{zd} \rceil$ solving nonlinear algebric constraint equations $f_i(\psi_{z1},...,\psi_{zd}) = 0$, $i = 1,2,...,d$ by numerical iteration Newton-Raphson (N-R) method.

In the initial configuration of mechanism the nonlinear algebric constraint equations $\bar{f}$ can be linearized by approximation with the sum of residual functions $\bar{f}_{(r)}$ obtained by introducing the arithmetic vector $\bar{\psi}_{z(r)}$ of estimated dependet global position coordinates (r is number of iteration step) and linear terms of Taylor series $\bar{f} \cong \bar{f}_{(r)} + V_{(r)} \Delta \bar{\psi}_{z(r)}$, where matrix $V_{(r)}$ is Jacobi matrix of the rank (d x d)

$V_{(r)} = \left[\frac{\partial f_i}{\partial \psi_{zj}} \right]_{(r)}$, $i = 1, 2, ..., m$, and $j = 1, 2, ..., d$, and $\Delta \bar{\psi}_{z(r)}$ is unknown arithmetic vector of corrections.
The arithmetic vector $\bar{f}$ of nonlinear algebric constraint equations is $\bar{f} = 0$ after introducing the solution $\bar{\psi}_z$, but $\bar{f} \neq \bar{f}_{(r)}$, $\bar{f}_{(r)} \neq 0$ (residuals) after introducing the arithmetic vector $\bar{\psi}_{z(r)}$ of estimated dependet global position coordinates. The arithmetic vector $\bar{\psi}_{z(r)}$ will be the accepted solution, if the norm $\|\bar{f}_{(r)}\|$ of residuals satisfy condition $\|\bar{f}_{(r)}\| \leq \varepsilon$, so maximum residual from all residuals is less than a specified tolerance ε. This can be achieved by iteration proces of converged numerical Newton-Raphson method
$\bar{\psi}_{z(r+1)} = \bar{\psi}_{z(r)} + \Delta \bar{\psi}_{z(r)}$, which will be finished, if the norm $\|\Delta \bar{\psi}_{z(r)}\|$ of corrections satisfy the condition $\|\Delta \bar{\psi}_{z(r)}\| \leq \varepsilon$ where ε is prescribed tolerance.

Reprezentácia modelu a metódy jeho analýzy v programe MSC.ADAMS
Analysis methods and model representation in ADAMS (T. J. Wielenga)

Aké sú základné kroky v algoritme riešenia sústav nelineárnych algebrických rovníc Newton – Raphsonovou iteračnou metódou a aké sú dôvody a výhody tohto postupu?

Zo sústavy rovníc $\overline{G}(\bar{x}) = 0$ uvažujeme na ilustráciu len jednu rovnicu $G(x) = 0$, ktorá má riešenie x_S, teda $G(x_S) = 0$.
Postup riešenia prebieha podľa nasledovných krokov:

1. krok — Urobíme odhad x_1 začiatočných hodnôt závislých premenných.

2. krok — Nelineárnu rovnicu $G(x) = 0$ aproximujeme prvými členmi Taylorovho radu. Podstatou tejto linearizácie je Newtonova metóda aproximácie krivky priamkou.
Sklon dotyčnice vyplýva z goniometrickej funkcie

$$tg\alpha = \frac{\Delta x}{\Delta G}$$

potom

$$G - G_1 = \frac{\partial G}{\partial x}(x - x_1)$$

V rovnici

$$-G_1(x_1) = \frac{\partial G}{\partial x}\Delta x_1$$

člen $-G_1(x_1)$ je zvyšok (reziduum)

$\frac{\partial G}{\partial x}$ je Jakobián

Δx_1 je korekcia

3. krok Faktorizácia Jakobiánu na trojuholníkové matice L, U

4. krok Výpočet korekcie Δx_1 doprednou a spätnou substitúciou

5. krok Nový krok iterácie $x_2 = x_1 + \Delta x_1$

Notes on Newton Rhapson Method

The Newton-Rhapson method is based upon the idea that the value of a function f(x) at x=b can be calculated if one knows the value of f(a) and the value of all the derivatives f'(a), f''(a), etc. This is known as a ***Taylor Series***

f(b) = f(a) + f'(a)(b-a) + f''(a)(b-a) ¸ +...

The goal of the Newton-Rhapson method is to find the root of a function, that is to find b, such that f(b)=0. Rather than computing all the derivatives, the series truncated after the first derivative and the approximation is repeated until the difference between two successive approximations is less than some small number. Thus one only needs to know the form of the function f(x) and its first derivative f'(x). Let the first approximation be at x= x0. Then, we can approximate the new value at x = x1 by

f(x1) = f(x0) + f'(x0)(x1- x0)

Now if we assume that f(x1) = 0, we can rewrite this as

0= f(x0) + f '(x0)(x1- x0)

By some straightforward algebra

x1= x0-(f(x0)/f'(x0))

or in general

xn+1= xn- (f(xn)/f'(xn))

This is repeated until the difference between two successive approximations is very small

abs((xn+1 -xn)/xn) < e

where e is some small number.
There are various constraints on convergence, including the fact that the function f(x) must be somewhat well behaved, however for our purposes, we can defer these questions to our mathematican friends.

Example

Find x, such that f*(x) = xł - 200 = 0* when x=b, i.e. the cube root of 200. The first derivative is f'(x) = 3x˛
making our first guess x0=5, f(5) = -75 and f'(5) = 75, so x1 = 5 - (-75/75) = 6.
Completing the problem using a spreadsheet.
n xn f(xn) f'(xn) xn+1 (xn+1 - xn)/xn
0 5.0 -75. 75. 6. 0.2
1 6.0 16. 108. 5.852 -2.5 E -03
2 5.852 0.391810 102.732 5.848 -6.5 E -04
3 5.848 2.55 E -04 102.599 5.848 -4.2 E -07
Thus the answer converges to 5.848. On my HP calculator to 7 digits the answer is 5.8480355.

Prínosy modifikovanej Newton-Raphsonovej iteračnej metódy.

http://math.fullerton.edu/mathews/n2003/NewtonAccelerateMod.html

http://ecs.fullerton.edu/~mathews/numerical.html

Kniha: Numerical Receipes in Fortran 77, Umenie vedeckých výpočtov

Analýza rýchlostí a zrýchlení bodov a členov VMS.

http://atc.sjf.stuba.sk/mechanika_vms_vypoctove_cvicenia.html