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Glossary for Engineering Mechanics II-Dynamics

2nd lecture: Kinematics and dynamics of a particle

Sections in 2nd lecture:

S1 Unit vectors of a body’s local position frame with origin moving along prescribed curve. Instantaneous velocity, tangential and normal accelerations of a body’s point.

S2 Euler’s equation for velocity of a point of rotating body.

S3 Dynamics of a particle. Newton’s laws. Principles of work-energy, linear and angular impulse-momentum for a particle. Relative and constrained motions of a particle. Free-body diagrams.

S1 Unit vectors of a body’s local position frame with origin moving along prescribed curve.

Triad $\bar{t}$, $\bar{n}$, $\bar{b}$ — Welding electrodes E_1, E_2 have to be oriented along main normal n of space trajectory of centre A of the effector.

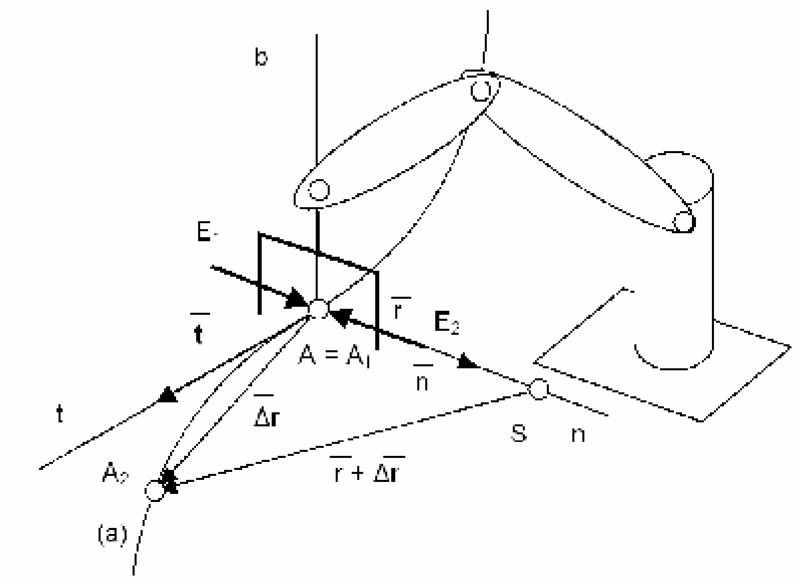

Fig.1L2 The triad of local coordinate system unit vectors $\bar{t}$, $\bar{n}$, $\bar{b}$ has origin A moving along space curve (a).

Velocity — The instantaneous velocity $\bar{v}$ of the end point A of position vector $\bar{r}$ can be derived as limit

$$\lim_{\Delta t \to 0} \frac{\Delta \bar{r}}{\Delta t} = \frac{d\bar{r}}{dt} = \dot{\bar{r}} = \bar{v} \tag{1S1}$$

Let we consider $s = \overset{\frown}{A_1 A_2}$ as curvilinear position coordinate of the point A_2 wrt the point A_1 and ds is a infinitesimal value

$$ds = \frac{\lim \Delta\bar{r}}{\Delta\bar{r} \to 0} \qquad (2S1)$$

then

$$\bar{v} = \frac{d\bar{r}}{dt} = \frac{d\bar{r}}{ds}\frac{ds}{dt} = \bar{t}\,v \qquad (3S1)$$

the unit tangential vector $\bar{t}$ and vector $\bar{K}$ of flexuosity are defined in the differential geometry

$$\frac{d\bar{r}}{ds} = \bar{t} \qquad (4S1)$$

$$\frac{d\bar{t}}{ds} = \bar{K} = K\,\bar{n}\,,\ \bar{K} = \frac{1}{R}\bar{n} \qquad (5S1)$$

Acceleration

The instantaneous acceleration $\bar{a}$ is rate of change in instantaneous velocity $\bar{v}$

$$\bar{a} = \frac{d\bar{v}}{dt} = \frac{d(\dot{s}\bar{t})}{dt} = \ddot{s}\,\bar{t} + \dot{s}\frac{d\bar{t}}{dt} = \bar{a}_t + \bar{a}_n \qquad (6S1)$$

$$\frac{d\bar{t}}{dt} = \frac{d\bar{t}}{ds}\frac{ds}{dt} = \bar{K}\,\dot{s} = \frac{1}{R}\bar{n}\,\dot{s}\,,$$

$$\bar{a}_t = \ddot{s}\,\bar{t} \qquad (7S1)$$

$$\bar{a}_n = \frac{\dot{s}^2}{R}\bar{n} \qquad (8S1)$$

In generally is $\bar{a} \neq \frac{dv}{dt} = a_t$ (9S1)

this is valid only for rectlinear motion

$$a_t = \frac{dv}{dt} = \frac{dv}{ds}\frac{ds}{dt} = \frac{d(v^2)}{2ds} \qquad (10S1)$$

$$a_t ds = v\,dv \qquad (11S1)$$

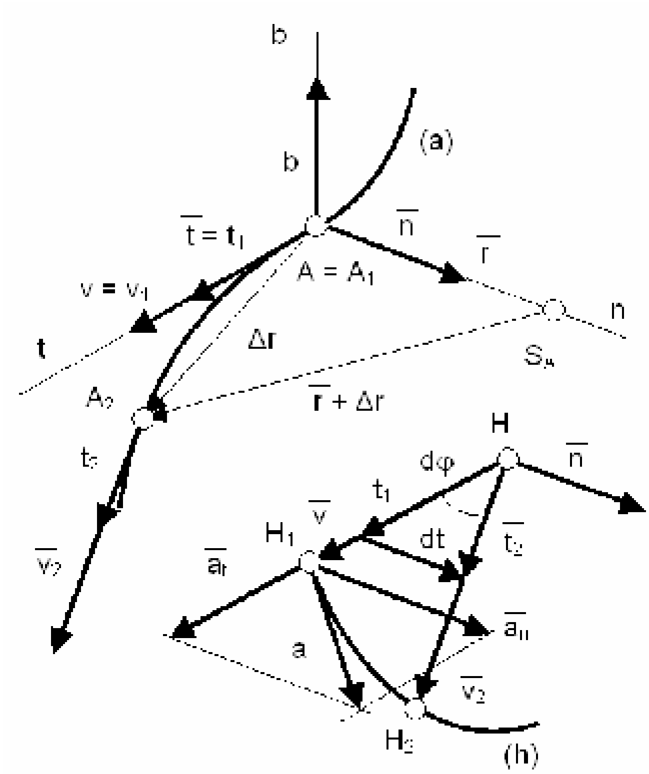

Fig.2L2 Line of action of s vector $\bar{a}$ of instantaneous acceleration is tangent to the hodoghraph (h) from end points H_i of velocities $\bar{v}_i$.

S2 Euler's equation for velocity of a point of rotating body

Rotational motion

The arm of welding robot represented by radius vector $\bar{r}$ rotates from initial position $\bar{r}_1$ to the subsequent position $\bar{r}_2$ about fixed axis with unit vector $\bar{e}$, so trajectory of end point A is a circle (a) and magnitude $|\bar{r}_1| = r$ and $|\bar{r}_2| = r$. When we consider an infinitesimal angle $\mathrm{d}j$ of slew of radius vector $\bar{r}$, then line of action of $\mathrm{d}\bar{r}$ becomes the tangent t perpendicular to the radius vector $\bar{r}$. The vector $\mathrm{d}\bar{j}$ perpendicular to the radius vector $\bar{r}$ represent the magnitude and orientation of angle $\mathrm{d}j$ of slew.

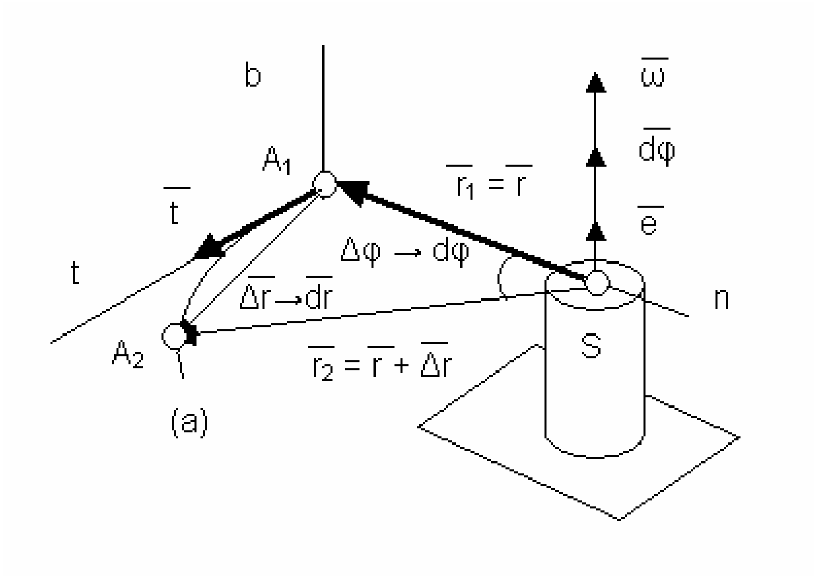

Fig.3L2 Rotation of a radius vector $\bar{r}$ about fixed axis.

Velocity

According to rule of vector cross product we can write

$$d\bar{r} = d\bar{j} \times \bar{r} \qquad (1S2)$$

Regarding that during infinitesimal small change dt of time the radius vector $\bar{r}$ as a finite quantity will remain without change, the time derivate of equation (1S2) is then in the form

$$\frac{d\bar{r}}{dt} = \frac{d\bar{j}}{dt} \times \bar{r} \qquad (2S2)$$

The time rate of change of the radius vector $\bar{r}$ is vector $\bar{v}$ of instantaneous velocity and time rate of change $d\bar{j}$ of the angular radius vector is vector $\bar{w}$ of instantaneous angular velocity of rotation of the radius vector $\bar{r}$, so from equation (2S2) we obtain a formula

$$\bar{v} = \bar{w} \times \bar{r} \qquad (3S2)$$

Equation (3S2) is known as Euler's equation for instantaneous circumferential velocity $\bar{v}$ of end point A of rotating radius vector $\bar{r}$.

Acceleration

The instantaneous acceleration $\bar{a}$ of end point A of rotating radius vector $\bar{r}$ can be derived as time derivate of cross product in the equation (3S2)

$$\bar{a} = \frac{d\bar{v}}{dt} = \frac{d(\bar{w} \times \bar{r})}{dt} = \frac{d\bar{w}}{dt} \times \bar{r} + \bar{w} \times \frac{d\bar{r}}{dt} \qquad (4S2)$$

time rate of change of the instantaneous angular velocity $\bar{w}$ is a vector $\bar{a}$ of instantaneous angular acceleration of rotation of the position vector $\bar{\mathrm{r}}$

$$\bar{\mathrm{a}} = \bar{a} \times \bar{\mathrm{r}} + \bar{w} \times \bar{\mathrm{v}} \qquad (5S2)$$

The first term in the (5S2) can be expressed in the form

$$\bar{a} \times \bar{\mathrm{r}} = a\,\bar{\mathrm{e}} \times \bar{\mathrm{r}} = a\,\mathrm{r}\bar{\mathrm{t}} = \mathrm{a_t}\bar{\mathrm{t}} \qquad (6S2)$$

Substituting $\bar{\mathrm{v}}$ in second term in the (5S2) from equation (3S2) we have to develop double cross product

$$\bar{w} \times (\bar{w} \times \bar{\mathrm{r}}) = \bar{w}\,(\bar{w}.\bar{\mathrm{r}}) - \bar{\mathrm{r}}\,(\bar{w}.\bar{w}) = -\bar{\mathrm{r}}w^2 = \mathrm{r}w^2\bar{\mathrm{n}} = \mathrm{a_n}\bar{\mathrm{n}} \qquad (7S2)$$

Substituting (6S2), (7S2) into equation (5S2) we obtain instantaneous acceleration $\bar{\mathrm{a}}$ of end point A of rotating radius vector $\bar{\mathrm{r}}$

$$\bar{\mathrm{a}} = a\,\mathrm{r}\bar{\mathrm{t}} + \mathrm{r}w^2\bar{\mathrm{n}} \qquad (8S2)$$

S3 Dynamics of a particle

Basic considerations

One of Newton's brilliant insights was that the same intuitive 'force' that causes deformation also causes change of motion state of mass, e.g. acceleration of mass. Force is related to deformation by material properties (elasticity, viscosity, etc.) and to motion by further laws of mechanics:

1) an mass object (particle or body) initially in rest or in motion with constant velocity tends to stay in initial motion state,
2) relation $\bar{\mathrm{F}} = \mathrm{m}\bar{\mathrm{a}}$ for a particle, and
3) the principle of action and reaction.

In words and informally, these laws of mechanics are

0) The laws of mechanics apply to any system (rigid or not):

a) Force and moment are *the* measure of mechanical interaction; and

b) Action = reaction with opposite direction applies to all interactions, ('every action has an equal and opposite reaction');

I) The net force on a system causes a net linear acceleration (*linear momentum balance*),

II) The net turning effect of forces on system causes it to rotationally accelerate (*angular momentum balance*), and

(III)The change of energy of a system is due to the energy flow into the system (*energy balance*).

When these ideas are supplemented with models of particular systems (e.g., of machines, buildings or human bodies) and with Euclidean geometry, they lead to predictions about the motions of these systems and about the forces which act upon them. Some of these results are classified into entire fields of research such as ‘fluid mechanics’, ‘vibrations’, ‘seismology’, ‘granular flow’, ‘biomechanics’, or ‘celestial mechanics’.The basic ideas in four laws of mechanics O-III also lead to other more mathematically advanced formulations of mechanics with names like ‘Lagrange’s equations’, ‘Hamilton’s equations’, ‘virtual work’, and ‘variational principles’.

Mass of a particle

Particle as a free body of mass m remains preferably in rest or moves straigthforward by constant velocity $\bar{v}$. Due to inertial property of its mass m the steady state of a free body changes by effect of other body acting directly by contact force in a common geometrical constraint or acting indirectly by its inherent force field (gravitational, magnetic, etc).

Gravitational force

Gravitational force of first mass is indirect steady action force acting on second mass as a result of gravitational field.

Inertia force

Inertia force is secondary reaction force againts change in velocity.

Linear momentum

Inertial property of body's mass m having instantaneous velocity $\bar{v}$ is expressed by concept of linear momentum $\bar{H} = m\bar{v}$ of a body.

Linear impulse

When force $\bar{F}$ is acting on body in a time t we denote by $\bar{I}_F$ the linear impulse $\bar{I}_F$ of force $\bar{F}$ and is

$$\bar{I}_F = \bar{F}\,t$$

The linear impulse $\bar{I}_F$ causes change $\Delta\bar{H}$ (increase) in linear momentum $\bar{H}$. The force $\bar{F}$ acting on body impart acceleration $\bar{a}$ to the body

$$\bar{F} = m\bar{a}$$

Total force

The total force $\bar{F}$ on a body is equal to rate of change of linear momentum $\bar{H}$

$$\bar{F} = \frac{d\bar{H}}{dt} = \dot{\bar{H}}$$

Constrained moment

Action force $\bar{F}$ acting at point C of a body constrained in a point A gives raise of reaction force $-\bar{F}$ in a geometrical constraint. The constrained couple of forces ($\bar{F}$, $-\bar{F}$) has rotational effect expressed by constrained moment $\bar{M}_A = \bar{r} \times \bar{F}$. The free couple of forces ($\bar{F}$, $-\bar{F}$) has rotational effect expressed by free moment $\bar{M}_A = \bar{r} \times \bar{F}$.

Circumferential velocity

Equation $\bar{v}_A = \bar{\omega} \times \bar{r}$ is known as Euler’s equation for computation of circumferential instantaneous velocity $\bar{v}_A$ of end

	point A of radius vector $\bar{r}$ rotating by instantaneous angular velocity $\bar{\omega}$.
Angular momentum	Angular momentum $\bar{K}_A$ of a body about a point A can be expressed as moment $\bar{K}_A = \bar{r} \times \bar{H}$ of linear momentum $\bar{H}$.
Angular impulse	When moment $\bar{M}$ is acting on body in a time t we denote by $\bar{I}_M$ the angular impulse $\bar{I}_M$ of force $\bar{F}$ and is $\bar{I}_M = \bar{M}\, t$. The angular impulse $\bar{I}_M$ causes change $\Delta\bar{K}$ (increase) in angular momentum $\bar{K}$. The moment $\bar{M}$ acting on body with moment of inertia J impart angular acceleration $\bar{\alpha}$ to the body $\bar{M} = J\,\bar{\alpha}$
Total moment	The total moment $\bar{M}$ on a body is equal to its rate of change of angular momentum $\bar{K}$ $\bar{M} = \dfrac{d\bar{K}}{dt} = \dot{\bar{K}}$
Kinetic energy	Kinetic energy is $E_K = m\, v^2$
Work	Work is $W = \bar{F} \,.\, \bar{r}$
Power	Power P is equal to rate of change of work W $P = \dfrac{dW}{dt} = \dot{W}$ also power P is equal to the sum of $P = \bar{F} \,.\, \bar{v} + \bar{M} \,.\, \bar{\omega}$
Potential energy	Potential energy is equal to the opposite value of potential U $E_P = -U$
Total energy	Total energy is equal to sum of kinetic and potential energy $E = E_K + E_P$

Couples of line vectors	Intergration of statics and dynamics (kinematics and kinetics) is presented by quantities expressed as couples of line vectors (resultant force $\bar{F}_A$, and resultant moment, or torque $\bar{M}_A$ acting at a body), (instataneous angular velocity $\bar{\omega}$ of a body, instataneous velocity $\bar{v}_A$ of a body's point A), (linear momentum $\bar{H}_A$ of a body, angular momentum $\bar{K}_A$ of a body as moment of linear momentum $\bar{H}_A$).