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Glossary

Engineering Mechanics II-Dynamics for bachelors study in 2nd year-clasis

1st lecture: Structural parameters

Sections in 1st lecture:

S1 The goal of the course Engineering Mechanics II-Dynamics

S2 Structural parameters of correct multibody systems (MBS)

S3 Incorrect MBS

S1 Engineering Mechanics

The goal of this course is to provide you with insights into how things work. Learning the laws of mechanics should help you will be able to make quantitative calculations that predict how things stand, move, and fall. Art and human insight, as opposed to precise algorithm or recipe is neccessary for integration of the applicability of computers for improved comprehension, rapid experimentation and genuine optimization of virtual prototypes with real complexity of properties of multibody systems (MBS).

Statics	The first part of the study of Engineering Mechanics is devoted to Statics, which is concerned with the equilibrium of bodies at rest or moving with constant velocity.
Dynamics	The second part of the study of Engineering Mechanics is devoted to Dynamics, which is concerned with bodies having accelerated motion.
Kinematics, kinetics	In this course, the subject of dynamics will be presented in two parts: kinematics, which treats only the geometric aspects of motion and kinetics, which is the analysis of the forces causing the motion.
Couples of line vectors	Intergration of statics and dynamics (kinematics and kinetics) is presented by quantities expressed as couples of line vectors (resultant force $\overline{F}_A$, and resultant moment, or torque $\overline{M}_A$ acting at a body), (instataneous angular velocity $\overline{\omega}$ of a body, instataneous velocity $\overline{v}_A$ of a body's point A), (linear momentum $\overline{H}_A$ of a body, angular momentum $\overline{K}_A$ of a body as moment $\overline{K}_A$ of linear momentum $\overline{H}_A$).
Mass of a body	Free body of mass m remains preferably in rest or moves straigthforward by constant velocity $\overline{v}$. Due to inertial property of its mass m the steady state of a free body changes by effect of other body acting directly by contact force in a common geometrical constraint or acting indirectly by its inherent force field (gravitational, magnetic, etc).
Constrained moment	Action force $\overline{F}$ acting at point C of a body constrained in a point A gives raise of reaction force $-\overline{F}$ in a geometrical constraint. The

constrained couple of forces $(\overline{F}, -\overline{F})$ has rotational effect expressed by constrained moment $\overline{M}_A = \overline{r} \times \overline{F}$.

Circumferential velocity — Equation $\overline{v}_A = \overline{\omega} \times \overline{r}$ is known as Euler's equation for computation of circumferential instantaneous velocity $\overline{v}_A$ of end point A of radius vector $\overline{r}$ rotating by instantaneous angular velocity $\overline{\omega}$.

Linear momentum — Inertial property of body's mass m having instantaneous velocity $\overline{v}$ is expressed by concept of linear momentum $\overline{H} = m\overline{v}$ of a body.

Angular momentum — Angular momentum $\overline{K}_A$ of a body about a point A can be expressed as moment $\overline{K}_A = \overline{r} \times \overline{H}$ of linear momentum $\overline{H}$.

S2 Structural parameters of correct MBS

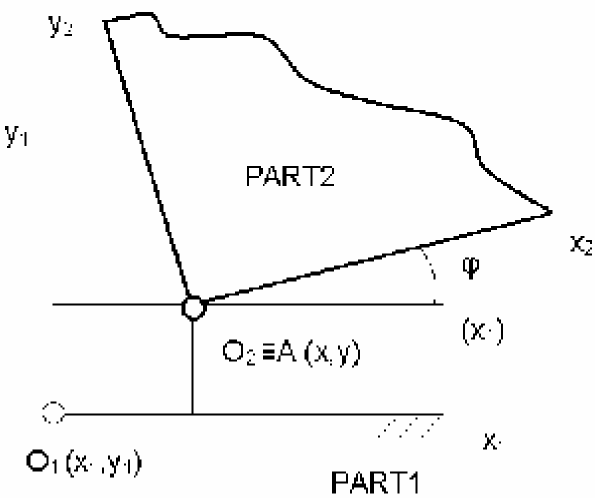

Fig.1 Position of PAR2 wrt PAR1 in plane

Position in plane — Unique position of the free, unconstrained body PAR2 (Fig.1) (later intended as link in a multibody system) wrt PAR1 in plane is given by two Cartesian position coordinates (x,y) of reference point A(x,y) identical with origin O_2 wrt origin O_1 of GCS (global coordinate system) and by variable (floating) angle $j = j_{12} = <(x_1, x_2)$ for angular displacement (slew) of PAR2 LCS (local coordinate system) axis x_2 wrt axis x_1 of GCS.

Position in space — To specify the position of the free (unconstrained) body P ≡ E (Fig.1) in the space wrt reference ground, it is necessary to define six mutually independent position coordinates $(x,y,z), (\varphi_x, \varphi_y, \varphi_z)$, because the general spatial motion of the free body can be replaced by the fictive translation represented by arbitrary chosen reference point with three Carthesian position coordinates (longitudinal displacements x,y,z) and by the fictive spherical motion with centre in this reference point represented by three position angles for sequence of the slews (for example angular displacements by Euler, resp. Cardan angles).

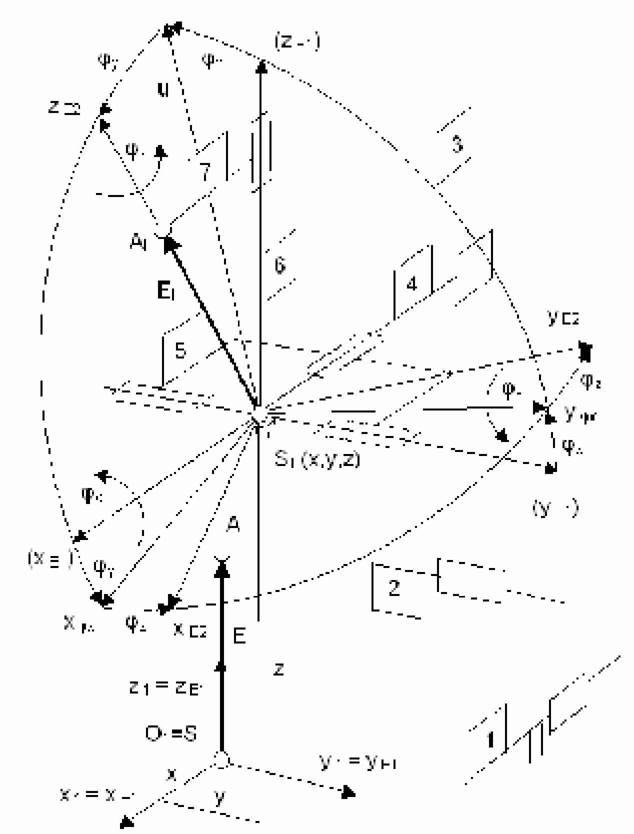

Fig.2 Position of the body $P \equiv E$ frame is determined by three Cartesian position coordinates (x,y,z) of it´s reference point S and Cardan´s angles $(\varphi_x, \varphi_y, \varphi_z)$ derived from given initial $P_I \equiv E_I$ and final $P_{II} \equiv E_{II}$ position of the body $P \equiv E$.

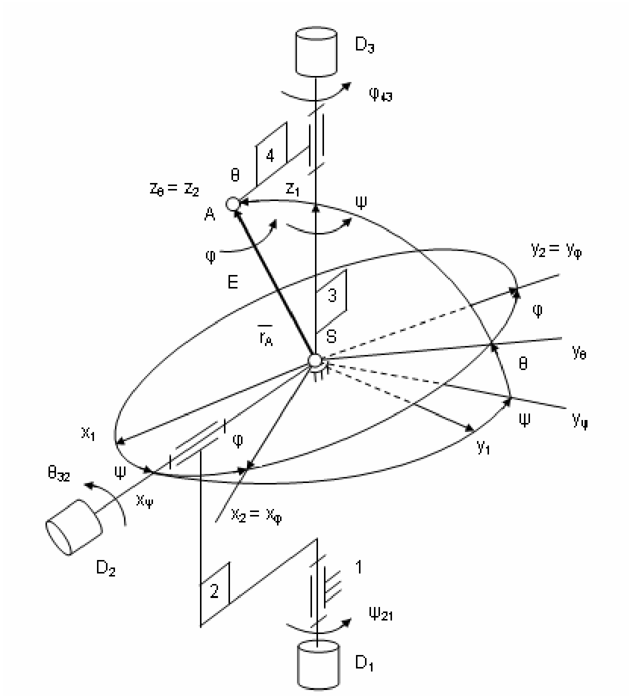

Fig.3 Initial and final position of body E frame determined by Euler´s angles *y* ,*q* ,*j* .

Mobility n_v — The number of mutually independent position coordinates is equal to the mobility n_v (or degrees of freedom DOF) of the free body, so the free body has mobility $n_v = 3$ in the plane and mobility $n_v = 6$ in the space.

Single joint — The single joint (holonomic geometrical constraint) is the assembly of pairing elements (surfaces, lines or points) of two links, which are held in contact by means of

- particular geometrical shapes (form closure),
- external forces (force closure),
- flexible materials (material closure).

Multiple joint — Multiple joint is connecting the number of v links, and $v > 2$ with total number p of all possible pairs of links, $p = \frac{v(v\text{-}1)}{2}$, from which number $(v\text{-}1)$ of pairs of links is independet.

Type t of the joint — The type t of the joint (geometrical constraint) is the number t, by which is reduced the mobility n_v of the former free link after its entry into this joint.
The type t of the joint (geometrical constraint) is also equal to the number of contact points, in which surfaces of adjacent links touch in corresponding generalized model introduced by Soni.

Notation of joints — Names (nomenclature) and shortcuts (acronyms) for joints:

- type $t=1$ (in space): general (G), plane-sphere (F_S), plane-peak (F_P), cylinder- cylinder (C_C),
- type $t=2$ (in space): sphere-groove (S_G), sphere-groove-helix (S_{GH}), plane-cylinder (F_C),
- type $t=3$ (in space): spherical (S), sphere-slotted-cylinder (S_{SC}), sphere-slotted-helix (S_{LH}), plane-plane (F_F),
- type $t=4$ (in space): torus-torus (T_T), sphere-slotted (S_L), cylindrical (C), plane-slipping surface (F_K),
- type $t=5$ (in space): revolute (R), prismatic (P), helical (H),

Multibody System

Multibody System MBS (assemblage, or constrained spatial system) is system of bodies (links) whose mutual movement is bounded by geometrical constraints (joints).

Number s_t

Number s_t is the number of geometrical constraints (joints) of the class t of all interconnected pairs of links in the multibody system

$s_t = \sum_{v=2}^{v_m} s_{tv}(v\text{-}1)$, where

s_{tv} is the number of jonts of the type t connecting the number v links

v_m is maximum number of joined links by one geometrical constraint in the multibody system.

Number s

Number of s is the total number of all interconnected pairs of links in the multibody system

$s = \sum_{t=1}^{t_m} s_t$, where

t_m is the maximum type t of the joint (geometrical constraint) in the multibody system

Type g of the link

The type g of the link is the number of adjacent links interconnected by this link. Unary link is of type $g=1$, binary link is of type $g=2$, ternary link is of type $g=3$.

Number u

Number u is the total number of all links in the multibody system

$u = \sum_{g=1}^{g_m} u_g$, where

u_g is the number of links of the type g in the multibody system

g_m is the maximum type g of the link in the multibody system.

Local mobility n_t

The local mobility n_t of the link in the joint of the type t is equal to the number of independent position coordinates of link wrt adjacent link in the joint and results from the subtraction $n_t = n_v - t$

Drawing

Construction drawing and production drawing is descriptive model for intended physical construction.

Kinematic diagram	Kinematic diagram (stick diagram, skeleton diagram) displays only essential skeleton of the physical construction of the multibody system.
Scetch	Scetch is more or less proportional kinematic diagram of the multibody system, but not exactly to scale.
Scaled diagram	Scaled diagram (metric model) is proportional kinematic diagram to the drawing, or physical construction of the multibody system.
Structural diagram	Structural diagram is scheme of structure (topological model) of the multibody system which does not contain metric data about dimensions and mutal configuration of links in space.
Unicomponential MBS	Each link in the unicomponential multibody system (assemblage) is connected to its neighbouring link by the geometrical contraint (joint).
Kinematic chain	Kinematic chain (KR) of the links is any unicomponential multibody system without frame.
Closed chain	Closed chain (UR) of the links has property $s = u$ with symbolic description of its structure by numbers 1234 (for example).
Kinematic loop	Kinematic loop (KS) of the links differs with closed chain (UR) only in the symbolic description of its structure by numbers 12341 (for example), where the first number is redoubled at the end of description.
Number k_s	Number k_s is number of all possible kinematic loops (KS) in the multibody system.
Open chain OR	Open chain (OR) has property $s = u - 1$
Open chain OROV	Open chain with joint elements at both tails (OROV) has property $s = u + 1$.
Single-loop chain	Single-loop chain (JR) has $k_s = 1$.
Multi-loop chain	Multi-loop chain (VR) has $k_s > 1$.
Combined chain	Combined chain (KR) has minimum one UR and one OR.
Basic chains	Basic chains ZJR and ZOROV are developed by decomposition of multi-loop chain (VR) into one ZJR and the number $k-1$ ZOROV, where k is the number of the basic kinematic loops ZKS.
Number k	The number k of the basic kinematic loops ZKS is according to the Euler invariant property related to the number s and u:

$$k = s - u + 1$$

Mobility n — Under the term mobility n of multibody system we mean the number of prescribed independet position coordinates of input (driving) links required to uniquely determine dependet position coordinates of driven output links

$$n = n_v (u - 1) - \sum_{t=1}^{t_m} t\, s_t$$

Multibody system — When one link in the kinematic chain is fixed (it becomes the frame) then arose the multibody system (assemblage).

Structure — Structure is multibody system (assemblage) with mobility $n \le 0$. If mobility of structure is $n = 0$, the structure is statically determinate, and in the case, when mobility of structure is $n < 0$, the structure is statically indeterminate.

Mechanism — Mechanism is multibody system (assemblage) with mobility $n \ge 1$. It is a mechanical device that has the purpose of transfering motion and/or force from a source (single input link if $n = 1$, or more input links if $n \ge 1$) to an ouput (single ouput link, or more links).

Types of mechanisms — According to type of structure of corresponding kinematic chain we analogically denote closed mechanism (UM), single-loop mechanism (JM), multi-loop mechanism (VM), open mechanism (OM) and combined mechanism (KM).

Linkage — Linkage is a mechanism which all joints are of the type $t = 5$ (space mechanisms), or $t = 2$ (planar mechanisms).

Cognate mechanisms — Geometrically different mechanisms are cognate, when they have the same transfer function.

Isomorphic diagrams — If different mechanisms have equal structural diagrams, these diagrams are isomorphic.

Coordinate systems — Each link (part) has local (own, or intrinsic) otrhonormal reference coordinate system.

Local coordinates — By the local position coordinates of the link (part) is described mutual local relative position of link wrt adjacent (neighbouring) link in the geometric constraint (joint). Local position coordinates can be in the form of

- variable (floating) abscissa $\bar{q}_{ij} = \overrightarrow{O_i O_j}$ for longitudinal displacement of part reference frames, and
- variable (floating) angle $\bar{\varphi}_{ij} = \mathbf{S}(x_i, x_j)$ for angular displacement of part reference frames.

Global coordinates

By the global position coordinates of the links in assemblages is described global relative position of links wrt frame (default is the part 1). Global position coordinates can be in the form of

- variable (floating) abscissa $\overline{p}_{1j} = \overrightarrow{O_1O_j}$ for longitudinal displacement of part reference frames, and
- variable (floating) angle $\bar{y}_{1j} = \mathbf{S}(x_1, x_j)$ for angular displacement of part reference frames.

Number c

Number c is total number of local position coordinates $q_i, i = 1, 2, ..., c$ of the links in the mechanism

$$c = \sum_{t=1}^{t_m} n_t s_t$$

and it is a sum $c = n + z$, where n is number of independet local position coordinates of the links (also n is mobility of mechanism) $q_{n\,i}$, $i = 1, 2, ..., n$, and

z is number of dependet local position coordinates of the links $q_{z\,i}$, $i = 1, 2, ..., z$.

Number z

Number z is number of dependet local position coordinates of the links

$$z = n_v k$$

Number m

Number m is total number of global position coordinates of the links

$$y_i, \; i = 1, 2, ..., m$$

and it is a sum $m = n + d$, where where n is number of independet global position coordinates of the links

$$y_{n\,i}, \; i = 1, 2, ..., n,$$

where n is mobility of mechanism and d is number of dependet global position coordinates of the links

$$y_{z\,i}, \; i = 1, 2, ..., d.$$

Number d

Number d of dependet global position coordinates of the links result from equation

$$d = 2k + s_1$$

Relation m and c

There is relation between m and c

$$m = c - k + s_1$$

Actual mobility n_S

If a multibody system (MBS) has in reality actual mobility n_S which is different as theoretical mobility n computed from formula $n = n_v(u-1) - \sum_{t-1}^{t_m} t s_t$, so $n_S \neq n$, then such MBS is called incorrect.

Correct MBS

A multibody system (MBS) with actual mobility $n_S = n$ equal to the theoretical mobility n computed from formula

$n = n_v(u-1) - \sum_{t-1}^{t_m} t s_t$ is called correct MBS. In a correct MBS each geometrical constraint of type t removes just the same number t DOF.

S3 Incorrect MBS Incorrect MBS has in reality actual mobility $n_S \neq n$ different as theoretical mobility n. The reason consist in fact, that formula $n = n_v(u-1) - \sum_{t-1}^{t_m} t s_t$ does not contain information neither about proportions (metrics) nor about mutual position (configurations) of links and geometrical joints. Theoretical mobility n of incorrect MBS may be zero (indicating a structure) or negative (indicating an indeterminate structure) but it can in reality, nevertheles, move, so its actual mobilty $n_s \geq 1$ due to special proportions (metrics) and mutual position (configurations) of links and geometrical joints.

Unremoved DOF Incorrect MBS has in reality actual mobility $n_S = n + n_N$ where n_N is number of unremoved DOF due to special proportions (metrics) and mutual position (configurations) of links and geometrical joints.

Singularities Under common term singularities in MBS we denote all reasons (passivity, redundancy, general constraint, irregularity,...) which causes that $n_S \neq n$, hence actual mobility n_S is different as theoretical mobility n.

Total passivity A constraint is totally passive if it can be removed and actual mobility of MBS does not change.

Partial passivity A constraint of a class t is partially passive if it remove from MBS only number n_O DOF, $n_O < t$.

Overconstrained MBS A MBS with theoretical mobility $n \leq 0$ and actual mobility $n_s \geq 1$ is overconstrained when actual mobility does not change after removing totally passive constraint.

Locked MBS If redundant constraint in MBS become inconsistent with other constraints (due manufacturing differences in link lengths or pivot locations), this causes that MBS will jam (locked).

Local mobility n_L Local mobility n_L is a passive (redundant) kinematic input which has no influence on the mobilty of output link.

Active mobility n_A Active mobility n_A is a active kinematic input which has influence on the mobilty of output link $n_A = n_S - n_L$.

Singular state	A MBS is in instantaneous singular state, when its links can displace with infinitely small values of position coordinates. If MBS is at permanent singular state, its links can displace with finite values of position coordinates.