2-5520 Theory of Mechanisms

Glossary

for bachelors study in 3rd year-classis, summer semester

Lecturer: Assoc. Prof. František Palčák, PhD., ÚAMM 02010

Lecture 9: Spherical motion of the body

Sections in Lecture 9:

S1 Spherical motion, Euler's angles for precession, nutation and local rotation. Open mechanism.

S2 Axoidal cones. Turning and centering acceleration of the point.

S3 Euler´s kinematic equation

S1 Spherical motion, Euler's angles for precession, nutation and local rotation.

Spherical motion — The local frame (O_2, x_2, y_2, z_2) of the body $E \equiv 2$ represented by the radius vector $\overline{r}_A$ (for example a joistick on Fig. 1) and ground reference frame (O_1, x_1, y_1, z_1) they have coincident origins $(O_2 \equiv O_1 \equiv S)$ during spherical motion of body 2 wrt ground 1. The local mobility $n_t = n_v - t$ of body $E \equiv 2$ in spherical joint of the class $t = 3$ is $n_3 = 3$ so the position of the body $E \equiv 2$ wrt ground 1 can be determined by three independent position coordinates (for example by the Euler´s angles y ,q ,j).

Euler angles — Let us consider that initial position of local frame $(O_2, x_2, y_2, z_2)_I$ of body E is coincident with ground frame (O_1, x_1, y_1, z_1). It is to determine three independent position coordinates (so-called 3-1-3 Euler´s angles y ,q ,j) by which is uniquely described final position $(O_2, x_2, y_2, z_2)_{II}$ of the body E. The line $x_\psi \equiv (x_1, y_1) \times (x_2, y_2)$ of intersection of both frame planes on Fig.1 provides us by angle $y \equiv (x_1, x_\psi)$, or $y \equiv (y_1, y_\psi)$, which should be applied as a first slew (precession) of local frame of body E about axis z_1. The angle $q \equiv (z_1, z_2)$ which yield from mutual position of axes z_1, z_2 or $q \equiv (y_\psi, y_\theta)$ is applied for a second slew (nutation) of local frame of body E about axis x_ψ. The local frame of body E will achieve it´s final position $(O_2, x_2, y_2, z_2)_{II}$ after application of third slew (spin) about axis z_2 by angle $y \equiv (x_\psi, x_2)$, or $y \equiv (y_\theta, y_2)$.

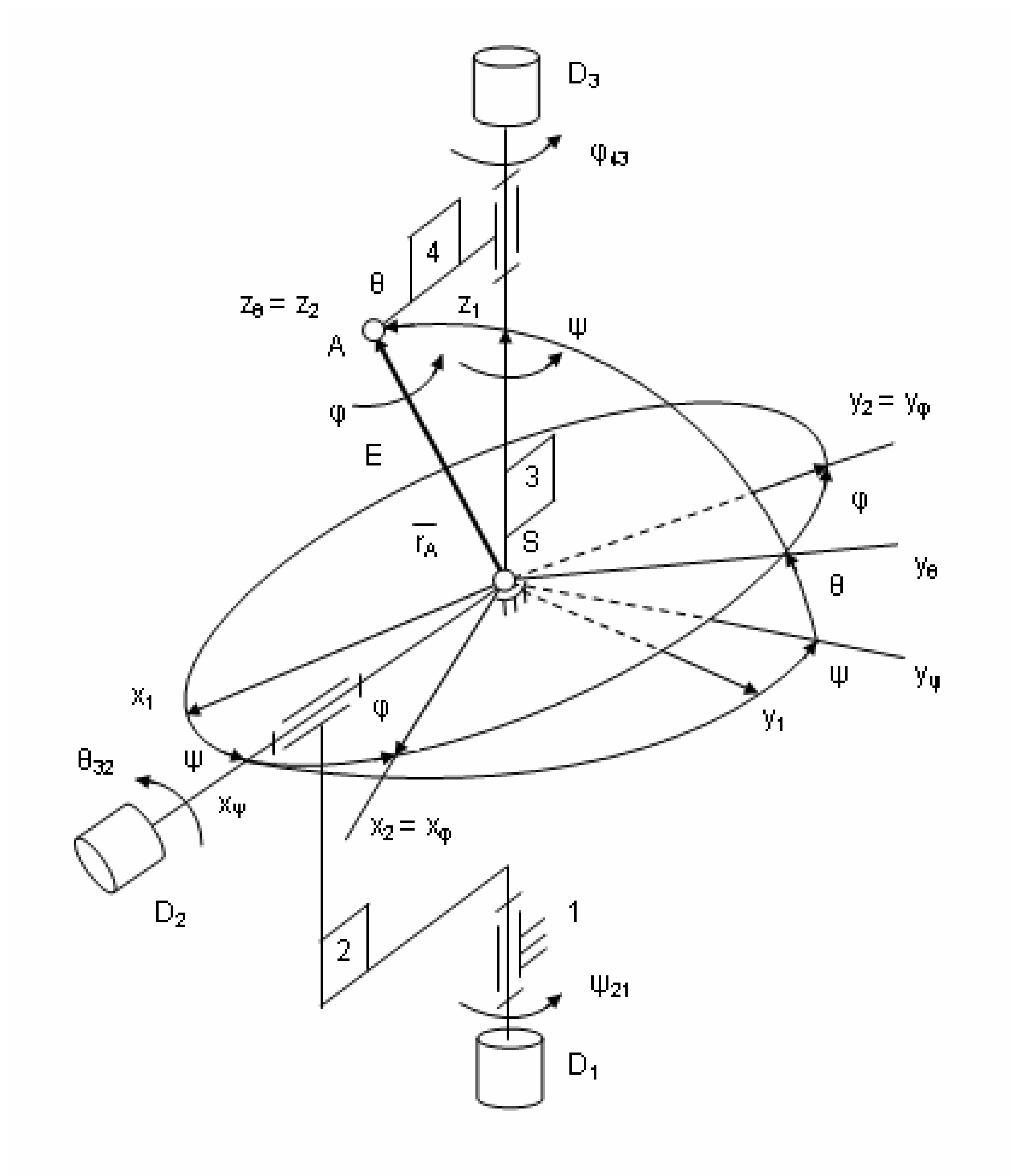

Fig.1 Initial and final position of body E frame determined by Euler´s angles ψ, θ, φ.

Open mechanism

Body E constrained to ground by spherical joint can be uniquely relocated from its initial first position of local frame $(O_2,x_2,y_2,z_2)_I$ into prescribed final second position $(O_2,x_2,y_2,z_2)_{II}$ by spherical open mechanism on Fig. 1, consisting from number $u = 4$ of links, number $s_3 = 3$ of revolute joints and with mobility $n = 3$. Part 4 of this open mechanism is carrying out spherical motion 4/1 equivalent from kinematics point of view with motion of part E. Three drivers D_1, D_2, D_3 realize subsequently or instantaneously three slews by angles of precession ψ_{21}, nutation θ_{32} and local rotation φ_{43} about axes permanently passing trough the centre S of spherical motion. Instantaneous angular velocity $\overline{\omega}$ expressed in local systems is the sum of angular velocities

$$\overline{\omega} = \dot{\psi}\,\overline{k}_{I} + \dot{\theta}\,\overline{i}_{\psi} + \dot{\varphi}\,\overline{k}_{II}$$

Using notation for open mechanism on Fig. 1 the average angular velocity $\overline{\omega}_{41}$ of spherical motion of fictive link E will be the sum of average angular velocities

$$\overline{\omega}_{41} = \overline{\dot{\psi}}_{43} + \overline{\dot{\theta}}_{32} + \overline{\dot{\varphi}}_{21}$$

S2 Centroidal cones. Turning and centering acceleration of the point.

Axoidal cones

The tip A of radius vector $\overline{r}_A$ on Fig. 2 displaces into near infinitesimal position by instantaneous slew of body E about axis o_ω of a rotation, line of action of instantaneous angular velocity $\overline{\omega}$ which is tangent of a movable axoidal cone c_2 rolling against fixed axoidal cone c_1 during spherical motion of body E wrt ground 1. Spherical motion occurs under condition $\overline{\omega} \times \overline{\alpha} \neq \overline{0}$ when line o_ω of action of instantaneous angular velocity $\overline{\omega}$ and line o_α of action of instantaneous angular acceleration $\overline{\alpha}$ are intersection lines.

Velocity $\overline{v}_A$

Instantaneous velocity of tip A of radius vector $\overline{r}_A$ is given by Euler´s equation $\overline{v}_A = \overline{\omega} \times \overline{r}_A$ like at rotational motion.

Acceleration $\overline{a}_A$

Instantaneous acceleration $\overline{a}_A = \overline{\alpha} \times \overline{r}_A + \overline{\omega} \times \overline{v}_A$ of a tip A of radius vector $\overline{r}_A$ can be obtained by time derivation of equation $\overline{v}_A = \overline{\omega} \times \overline{r}_A$ but unlike rotational motion there are skew lines of action for of instantaneous angular velocity $\overline{\omega}$ and instantaneous angular acceleration $\overline{\alpha}$. So instead tangential and normal components, the first term $\overline{a}_{\alpha A} = \overline{\alpha} \times \overline{r}_A$ is denoted as turning acceleration and second term $\overline{a}_{\omega A} = \overline{\omega} \times \overline{v}_A$ is denoted as centering acceleration of the point A.

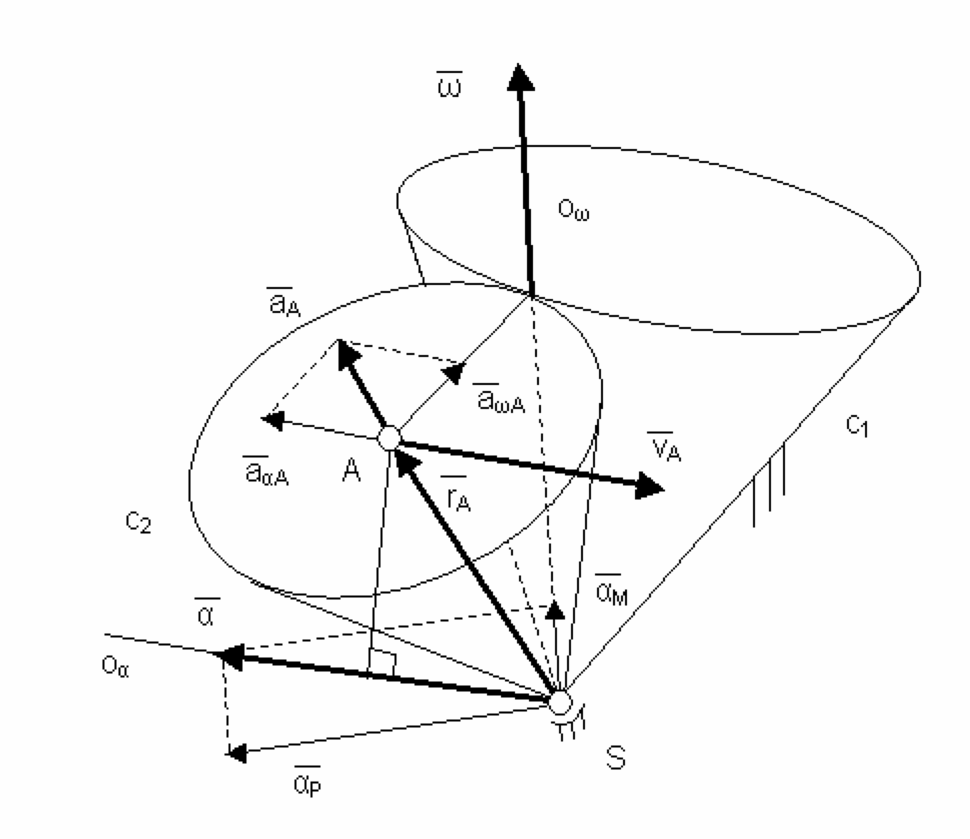

Fig. 2 Instantaneous angular velocity $\bar{\omega}$ and angular acceleration $\bar{\alpha}$ of a spherical motion of radius vector $\bar{r}_A$.

Projections $\bar{\alpha}_M, \bar{\alpha}_P$

Instantaneous angular acceleration $\bar{\alpha}$ has projection $\bar{\alpha}_M$ into line o_ω of action of instantaneous angular velocity $\bar{\omega}$ and perpendicular projection $\bar{\alpha}_P$, so $\bar{\alpha} = \bar{\alpha}_M + \bar{\alpha}_P$. Projection $\bar{\alpha}_M$ causes time change of angular velocity $\bar{\omega}$ magnitude and perpendicular projection $\bar{\alpha}_P$ changes orienation of angular velocity $\bar{\omega}$.

S3 Euler´s kinematic equation

Euler´s kin. eq.

For purpose to express instantaneous velocity $\bar{v}_A$ and acceleration of tip A of body E it is necessary to transform its instantaneous angular velocity $\bar{\omega} = \dot{\psi}\bar{k}_I + \dot{\vartheta}\bar{i}_\psi + \dot{\varphi}\bar{k}_{II}$ in the local system $(O_2,x_2,y_2,z_2)_{II}$

$$\bar{\omega}_E = \omega_x \bar{i}_{II} + \omega_y \bar{j}_{II} + \omega_z \bar{k}_{II}$$

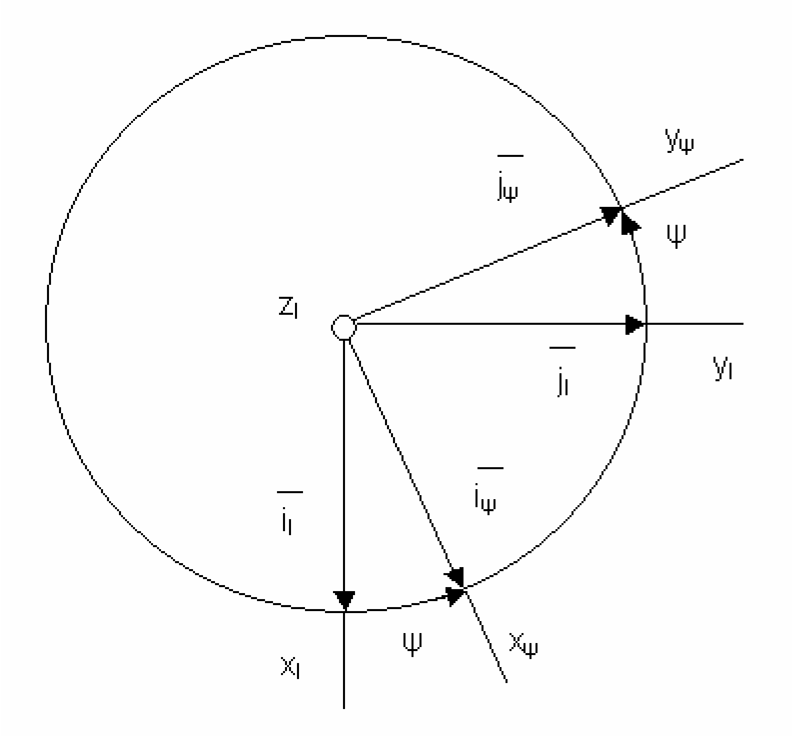

Fig. 3 Relationships between unit vectors $\overline{i}_I, \overline{j}_I, \overline{i}_\psi, \overline{j}_\psi$ for slew angle ψ

Relationships between unit vectors can be obtained by their projections during all three slews by Euler´s angles ψ, θ, φ like

$$\overline{i}_I = \left(\overline{i}_I . \overline{i}_\psi\right) \overline{i}_\psi + \left(\overline{i}_I . \overline{j}_\psi\right) \overline{j}_\psi$$

on Fig. 3 which yields to the form

$$\overline{i}_I = c\psi\, \overline{i}_\psi - s\psi\, \overline{j}_\psi$$

Euler´s kinematic equation is used in the form

$$\overline{\omega}_E = (\dot{\psi}\, s\theta\, s\varphi + \dot{\theta}\, c\varphi)\, \overline{i}_{II} + (\dot{\psi}\, s\theta\, c\varphi - \dot{\theta}\, s\varphi)\, \overline{j}_{II} + (\dot{\psi}\, c\theta + \dot{\varphi})\, \overline{k}_{II}$$