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2-5532 Theory of Mechanisms

Applied Mechanics and Mechatronics for bachelor study, year 3, summer sem.
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Lecture 5: Velocities during simultaneous motions

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S1 Resulting angular velocity

Angular velocities

The position vector $\overline{r}_{B31}$ of the point B_{31} from PAR3 has components expressed in space $\{1\}$. Let us differentiate vector $\overline{r}_{B31}$ with respect to time in different spaces $\{2\}$, and $\{3\}$

$$[\overline{r}_{B31}]_1^{\bullet}=[\overline{r}_{B31}]_3^{\bullet}+\overline{w}_{31}\times\overline{r}_{B31} \quad (1)$$

$$[\overline{r}_{B31}]_1^{\bullet}=[\overline{r}_{B31}]_2^{\bullet}+\overline{w}_{21}\times\overline{r}_{B31} \quad (2)$$

$$[\overline{r}_{B31}]_2^{\bullet}=[\overline{r}_{B31}]_3^{\bullet}+\overline{w}_{32}\times\overline{r}_{B31} \quad (3)$$

Comparing (1), (2) and and substituting (3) we obtain

$$\overline{w}_{31}\times\overline{r}_{B31}=\overline{w}_{32}\times\overline{r}_{B31}+\overline{w}_{21}\times\overline{r}_{B31} \quad (4)$$

From equation (4) yield that resulting instantaneous angular velocity $\overline{w}_{31}$ of the of general planar motion 3/1 of the PAR3 (coupler in the piston–crank mechanism) wrt PAR1 in mechanism can be expressed as sum

$$\overline{w}_{31}=\overline{w}_{32}+\overline{w}_{21} \quad (5)$$

of instantaneous angular velocities $\overline{w}_{32},\overline{w}_{21}$. It is the decomposition of general planar motion 3/1 of the PAR 3 wrt PAR1 (ground) when PAR3 is displaced from its initial position $\overrightarrow{A_1B_1}$ to the final position $\overrightarrow{A_2B_2}$ by fictive subsequent, or real simultaneous rotations 2/1 and 3/2. So general planar motion 3/1 of the PAR3 can be decomposed to the carrying motion 2/1 and local relative motion 3/2.
The instantaneous angular velocity $\overline{w}_{31}$ in the equation (5) we denote as the instantaneous resulting angular velocity

and it is the sum of instantaneous local relative angular velocity $\bar{\omega}_{32}$ and instantaneous carrying angular velocity $\bar{\omega}_{21}$.

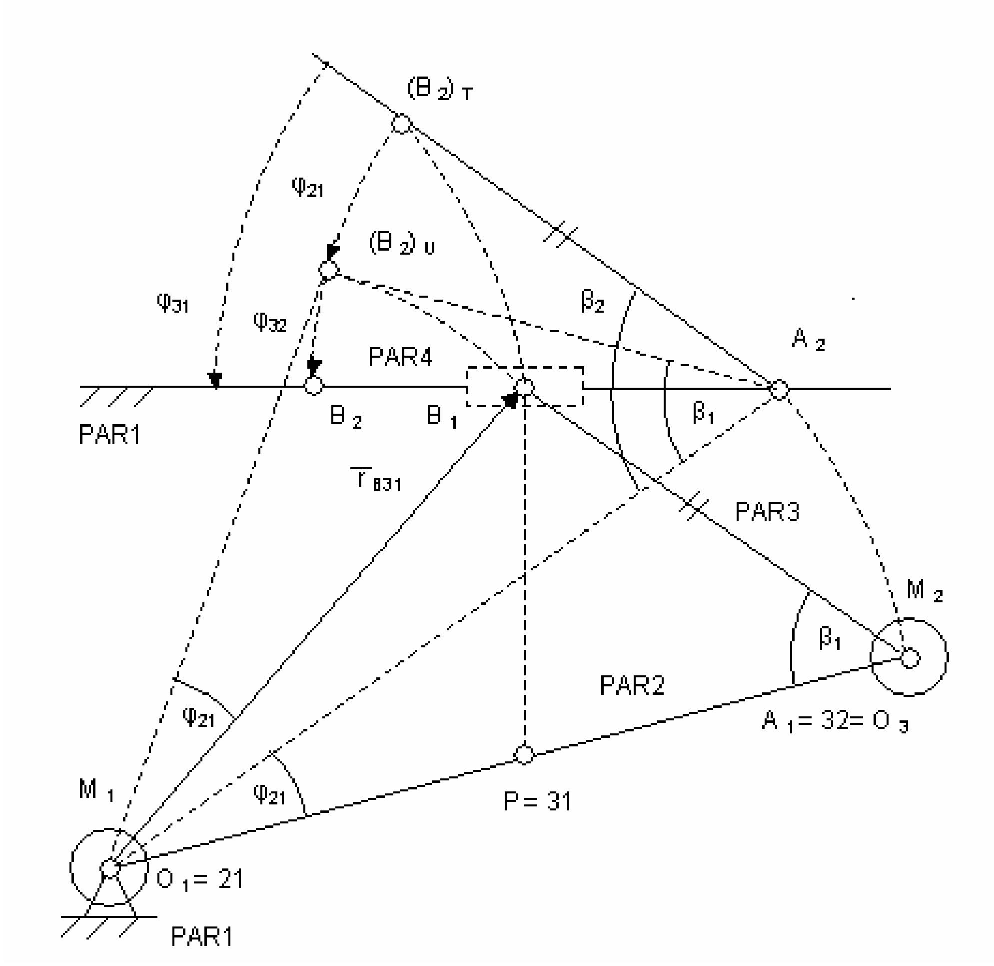

Fig.1 The open mechanism with arm PAR2 actuated by motor M1 and arm PAR3 actuated by motor M2.

S2 Simultaneous motions

Simultaneous motions

General plane motion 3/1 of the PAR3 can be decomposed to the fictive carrying motion 2/1 and fictive local relative motion 3/2.

Realization of this decomposition is demonstrated by open mechanism on Fig. 1 with arm PAR2 actuated by motor M1 and arm PAR3 actuated by motor M2. The PAR3 can be displaced from its initial position $\overrightarrow{A_1B_1}$ to the final position $\overrightarrow{A_2B_2}$ by subsequent, or simultaneous rotations 2/1 and 3/2.

During fictive carrying motion 2/1 about point O_1 by a finite angle φ_{21} the PAR3 is fixed to PAR2 $(3 \equiv 2)$ and PAR3 is displaced from its initial position $\overrightarrow{A_1B_1}$ to the the position $\overrightarrow{A_2(B_2)_U}$.

During fictive local relative rotation 3/2 the PAR3 is displaced about point O_3 by a finite angle j_{32} from the position $\overrightarrow{A_2(B_2)_U}$ to the final position $\overrightarrow{A_2B_2}$.
For constant angular velocity $w = \mathrm{const.}$ of rotation the slew angle is $j = w\,t$ and considering the time $t = 1$ s is also $j = w$. Then resulting finite angle is the sum

$$j_{31} = j_{32} + j_{21} \tag{6}$$

and corresponding fictive constant angular velocities, by which arms PAR2 and PAR3 in open mechanism rotates, are

$$\bar{w}_{31F} = \bar{w}_{32F} + \bar{w}_{21F} \tag{7}$$

The fictive constant angular velocities from Eq. (7) becomes actual angular velocities as are in the Eq. (5) for infinitesimal small angles in the Eq. (6).
The instantaneous angular velocity $\bar{w}_{31}$ in eq. (5) we denote as the instantaneous resulting angular velocity and it is the sum of instantaneous local relative angular velocity $\bar{w}_{32}$ and instantaneous carrying angular velocity $\bar{w}_{21}$.

S3 Resal's angular acceleration and resulting angular acceleration

Angular accelerations

Resulting angular acceleration $\bar{a}_{31}$ we obtain via time derivative of equation (5)

$$[\bar{w}_{31}]_1^{\bullet} = [\bar{w}_{32}]_1^{\bullet} + [\bar{w}_{21}]_1^{\bullet} \tag{8}$$

By direct time derivative we obtain $[\bar{w}_{31}]_1^{\bullet} = \bar{a}_{31}$ (9)
and time derivative of $\bar{w}_{32}$ in different space $\{1\}$ is

$$[\bar{w}_{32}]_1^{\bullet} = [\bar{w}_{32}]_2^{\bullet} + \bar{w}_{21} \times \bar{w}_{32} \tag{10}$$

where $[\bar{w}_{32}]_2^{\bullet} = \bar{a}_{32}$ (11)

The cross product $\bar{w}_{21} \times \bar{w}_{32}$ is known as Resal's angular acceleration

$$\bar{a}_R = \bar{w}_{21} \times \bar{w}_{32} \tag{12}$$

Substituting (9) – (12) into (8) we obtain equation for resulting angular acceleration

$$\bar{a}_{31} = \bar{a}_{32} + \bar{a}_{21} + \bar{a}_R \tag{13}$$