Institute of Applied Mechanics and Mechatronics,
Faculty of Mechanical Engineering STU Bratislava; http://www.sjf.stuba.sk

2-5532 Theory of Mechanisms

Applied Mechanics and Mechatronics for bachelor study, year 3, summer sem.
Guarantee: Assoc.Prof. František Palčák, PhD., ÚAMM 02010

Lecture 3: Triad TNB and rotational motion

Sections in Lecture 3:

S1 Triad TNB of local coordinate system unit vectors with origin moving along space curve

S2 Rotational motion

S1 The triad $\bar{t}$, $\bar{n}$, $\bar{b}$ of local coordinate system unit vectors with origin A moving along space curve

Triad $\bar{t}$, $\bar{n}$, $\bar{b}$

Welding electrodes E_1, E_2 have to be oriented along main normal n of space trajectory of centre A of the effector.

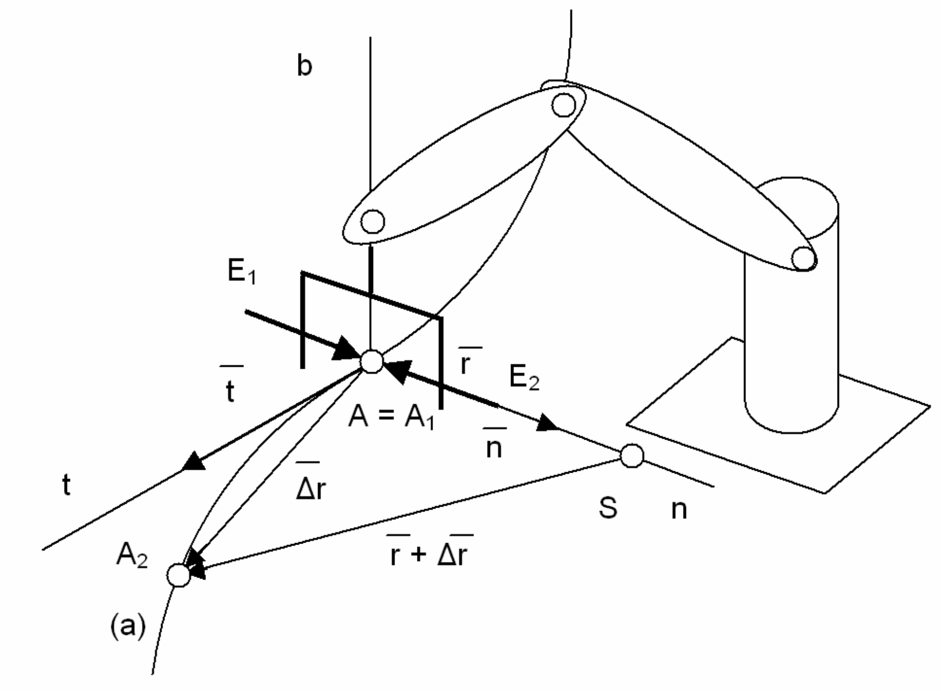

Fig.1 Triad $\bar{t}$, $\bar{n}$, $\bar{b}$ of local coordinate system unit vectors with origin moving along space curve

Velocity

The instantaneous velocity $\bar{v}$ of the tip point A of position vector $\bar{r}$ can be derived as limit

$$\lim_{\Delta t \to 0} \frac{\Delta \bar{r}}{\Delta t} = \frac{d\bar{r}}{dt} = \dot{\bar{r}} = \bar{v}$$

Let us consider $s = \overset{\frown}{A_1A_2}$ as curvilinear position coordinate of the point A_2 wrt the point A_1 and ds is an infinitesimal value

$$ds = \frac{\lim \Delta\bar{r}}{\Delta\bar{r} \to 0}$$

then

$$\bar{v} = \frac{d\bar{r}}{dt} = \frac{d\bar{r}}{ds}\frac{ds}{dt} = \bar{t}\, v$$

the unit tangential vector $\bar{t}$ and vector $\bar{K}$ of flexuosity are defined in the differential geometry

$$\frac{d\bar{r}}{ds} = \bar{t}\ , \ \frac{d\bar{t}}{ds} = \bar{K} = K\,\bar{n}\ , \ \bar{K} = \frac{1}{R}\bar{n}$$

Acceleration

The instantaneous acceleration is

$$\bar{a} = \frac{d\bar{v}}{dt} = \frac{d(\dot{s}\bar{t})}{dt} = \ddot{s}\bar{t} + \dot{s}\frac{d\bar{t}}{dt} = \bar{a}_t + \bar{a}_n$$

$$\frac{d\bar{t}}{dt} = \frac{d\bar{t}}{ds}\frac{ds}{dt} = \bar{K}\dot{s} = \frac{1}{R}\bar{n}\dot{s}$$

$$\bar{a}_t = \ddot{s}\bar{t}\ , \bar{a}_n = \frac{\dot{s}^2}{R}\bar{n}$$

In general is $\bar{a} \neq \frac{dv}{dt} = a_t$ this is valid only for rectilinear motion

$$a_t = \frac{dv}{dt} = \frac{dv}{ds}\frac{ds}{dt} = \frac{d(v^2)}{2ds}$$

$$a_t ds = v\, dv$$

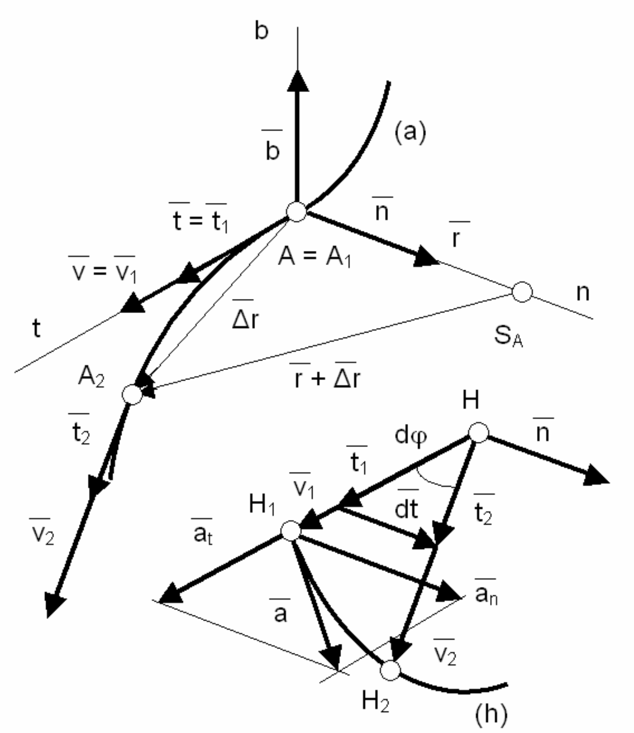

Fig.2 Hodograph (h) as locus of tip points of velocities

S2 Rotational motion

Rotational motion

The arm of welding robot represented by position vector $\overline{r}$ rotates from initial position $\overline{r}_1$ to the subsequent position $\overline{r}_2$ about fixed axis with unit vector $\overline{e}$, so trajectory of end point A is a circle (a) and magnitude $|\overline{r}_1| = r$ and $|\overline{r}_2| = r$. When we consider an infinitesimal angle $\mathrm{d}j$ of slew of position vector $\overline{r}$, then line of action of $\mathrm{d}\overline{r}$ becomes the tangent t perpendicular to the position vector $\overline{r}$ The vector $\mathrm{d}\overline{j}$ perpendicular to the position vector $\overline{r}$ represents the magnitude and orientation of angle $\mathrm{d}j$ of slew.

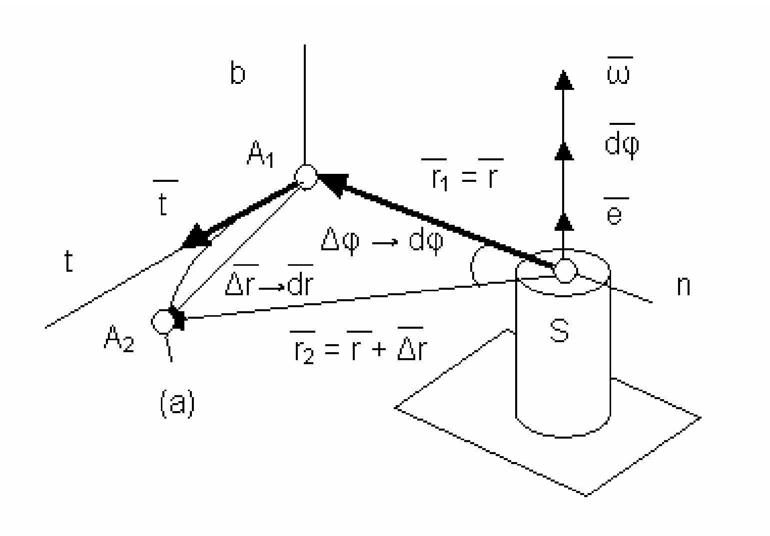

Fig.3 The welding robot with rotating arm

Velocity

According to rule of vector cross product we can write

$$\mathrm{d}\bar{\mathrm{r}} = \mathrm{d}\bar{j} \times \bar{\mathrm{r}} \tag{1}$$

Regarding that during infinitesimal small change dt of time the position vector $\bar{\mathrm{r}}$ as a finite quantity will remain without change, the time derivate of equation (1) is then in the form

$$\frac{\mathrm{d}\bar{\mathrm{r}}}{\mathrm{dt}} = \frac{\mathrm{d}\bar{j}}{\mathrm{dt}} \times \bar{\mathrm{r}} \tag{2}$$

The time rate of change of the position vector $\bar{\mathrm{r}}$ is vector $\bar{\mathrm{v}}$ of instantaneous velocity and time rate of change of the angular position vector $\mathrm{d}\bar{j}$ is vector $\bar{w}$ of instantaneous angular velocity of rotation of the position vector $\bar{\mathrm{r}}$, so from equation (2) we obtain a formula

$$\bar{\mathrm{v}} = \bar{w} \times \bar{\mathrm{r}} \tag{3}$$

Equation (3) is known as Euler's equation for instantaneous velocity $\bar{\mathrm{v}}$ of end point A of rotating radius vector $\bar{\mathrm{r}}$.

Acceleration

The instantaneous acceleration can be derived as time derivate of cross product in the equation (3)

$$\bar{\mathrm{a}} = \frac{\mathrm{d}\bar{\mathrm{v}}}{\mathrm{dt}} = \frac{\mathrm{d}(\bar{w} \times \bar{\mathrm{r}})}{\mathrm{dt}} = \frac{\mathrm{d}\bar{w}}{\mathrm{dt}} \times \bar{\mathrm{r}} + \bar{w} \times \frac{\mathrm{d}\bar{\mathrm{r}}}{\mathrm{dt}} \tag{4}$$

time rate of change of the instantaneous angular velocity $\bar{w}$ is a vector $\bar{a}$ of instantaneous angular acceleration of rotation of the position vector $\bar{\mathrm{r}}$

$$\bar{\mathrm{a}} = \bar{a} \times \bar{\mathrm{r}} + \bar{w} \times \bar{\mathrm{v}} \tag{5}$$

The first term in the (5) can be expressed in the form

$$\bar{a} \times \bar{\mathrm{r}} = a\,\bar{\mathrm{e}} \times \bar{\mathrm{r}} = a\,\mathrm{r}\bar{\mathrm{t}} = \mathrm{a_t}\,\bar{\mathrm{t}} \tag{6}$$

Substituting $\bar{\mathrm{v}}$ in second term in the (5) from equation (3) we have to develop double cross product

$$\bar{w} \times (\bar{w} \times \bar{\mathrm{r}}) = \bar{w}\,(\bar{w}.\bar{\mathrm{r}}) - \bar{\mathrm{r}}\,(\bar{w}.\bar{w}) = -\bar{\mathrm{r}}w^2 = \mathrm{r}w^2\bar{\mathrm{n}} = \mathrm{a_n}\bar{\mathrm{n}} \tag{7}$$

Substituting (6), (7) into equation (5) we obtain instantaneous acceleration $\bar{\mathrm{a}}$ of end point A of rotating radius vector $\bar{\mathrm{r}}$

$$\bar{\mathrm{a}} = a\,\mathrm{r}\bar{\mathrm{t}} + \mathrm{r}w^2\bar{\mathrm{n}} \tag{8}$$