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# 2-5532 Theory of Mechanisms

Applied Mechanics and Mechatronics for bachelor study, year 3, summer sem.
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## Lecture 2: Principle of rolling centrodes.

**Sections in Lecture 2:**



### S1 The finite angle of slew

Types of motions — In the slider crank mechanism on Fig.1 the PAR2 (crank) wrt PAR1 (ground) rotates 2/1, the PAR4 (piston) wrt PAR1 (ground) translates 4/1, and the PAR3 (coupler) moves wrt PAR1 (ground) via general planar motion 3/1 (it perform neither rotation, nor translation).

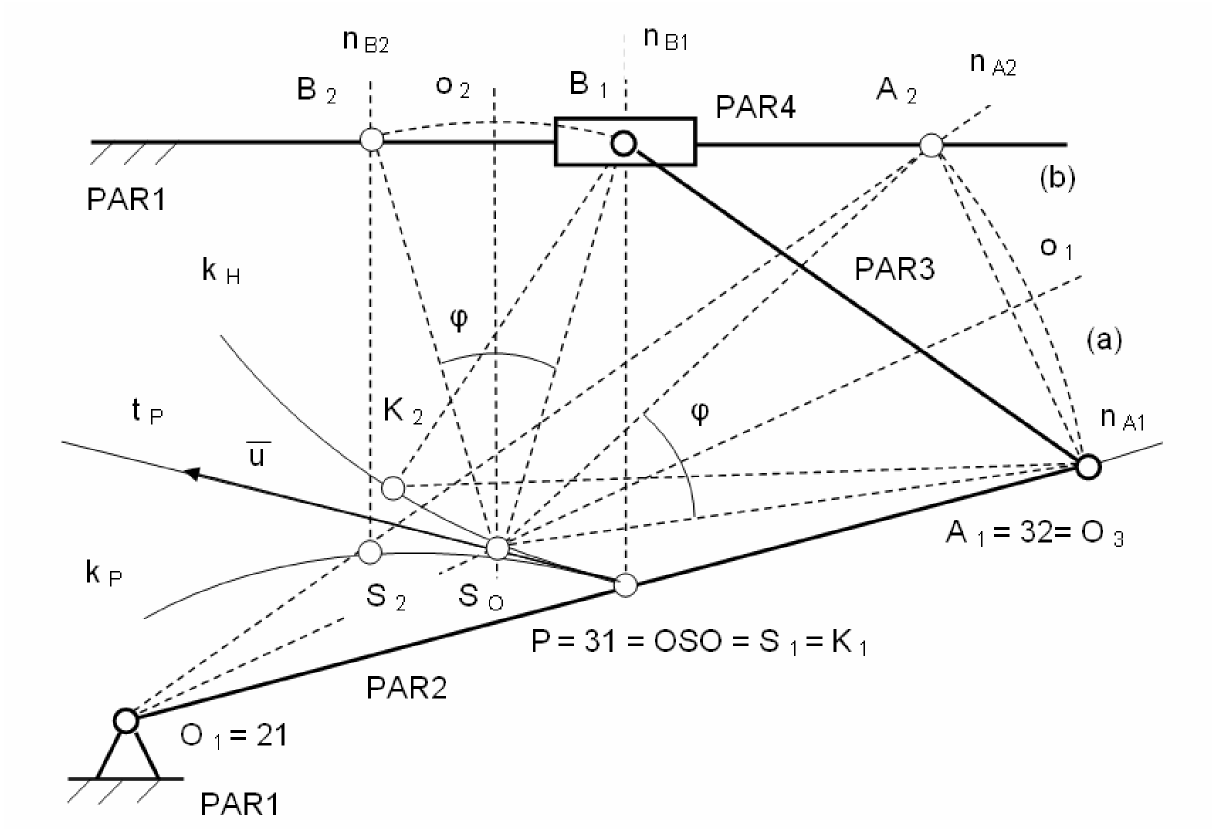


Fig.1 The slider crank mechanism in initial configuration ($O_1A_1B_1$) and final configuration ($O_1A_2B_2$).

Finite angle of slew — The lamina with coupler $\overrightarrow{AB}$ can be displaced from its initial position $\overrightarrow{A_1B_1}$ to the final position $\overrightarrow{A_2B_2}$ via slew about center

$S_o = o_1 \times o_2$ (bisection of abscisses) with finite angle $j =\angle(A_1S_oA_2) =\angle(B_1S_oB_2)$.

Instantaneous centre — If the finite angle $j$ of slew will be reduced to become infinite small, then the lamina with coupler $\overrightarrow{AB}$ will be displaced from its initial position $\overrightarrow{A_1B_1}$ to the infinitesimal close position via slew about intersection point $S_1$ of normals $n_{A1}, n_{B1}$ to the actual paths $(a),(b)$.
This intersection point $S_1$ is instantaneous slew center $S_1 = (OSO_{31})_1 = n_{A1} \times n_{B1} = P$ of zero velocity of coupler PAR3 wrt PAR1. For configuration of mechanism when coupler is in the position $\overrightarrow{A_2B_2}$ the adjacent intersection point $S_2 = (OSO_{31})_2 = n_{A2} \times n_{B2}$ is new instantaneous slew centre $S_2$.

## S2 Control curves – rolling centrodes

Fixed centrode — The locus of the instantaneous slew centres $\{S_i\}$ of zero velocity traced on the fixed lamina during general planar motion 3/1 of the coupler PAR3 wrt PAR1 is called the fixed centrode $k_P$.

Movable centrode — When the triangle $\Delta(A_2B_2S_2)$ will be repositioned to the initial position $\overrightarrow{A_1B_1}$ the point $S_2$ from movable lamina of coupler PAR 3 becomes $K_2$. After generalization of this procedure $\Delta(A_iB_iS_i) \rightarrow \Delta(A_1B_1K_i)$ the locus of the instantaneous slew centres $\{K_i\}$ in the movable plane is called the movable centrode $k_H$.

Control curves — The general planar motion 3/1 of the coupler PAR3 wrt PAR1 can be replaced by pure rolling of movable centrode $k_H$ against fixed centrode $k_P$ (control curves).

Center of curvature — The center $S_A$ of curvature of point $A$ trajectory $(a)$ is the center of osculating circle, by which is trajectory $(a)$ replaced in the neighborhood of point $A$. When trajectory $(a)$ is a circle then center $S_A$ of curvature is coincident with center of this circle. When trajectory $(a)$ is a straight line, then center $S_A$ of curvature is a step point. During steady rotation or translation is center $S_A$ of curvature identical with instantaneous slew centre $OSO$. In case of general plane motion center $S_A$ of curvature and instantaneous slew center $OSO$ are different points.

## S3 Application

**Application**

Hypocyclic gear train on Fig.2 is one of well known appliance of control curves, where the movable centrode $k_H$ is planet wheel circle with radius $R_P$ and the fixed centrode $k_P$ is sun wheel circle with radius $R_C$. If the transmission ratio $m_{PC}$ is defined by ratio of pitch circles radii $m_{PC} = \frac{R_P}{R_C} = \frac{1}{3}$, then point $C_1$ on pitch circle of a planet generates trajectory named Steiner's hypocycloid which is approximately of circle shape. The piston in dwell mechanism is required to be in the rest during phase of coupler BC revolution about point B, when point C is moving from $C_1$ to the $C_2$. The requirement is to minimize difference DX in position of end point E from coupler and reference point R from the ground.

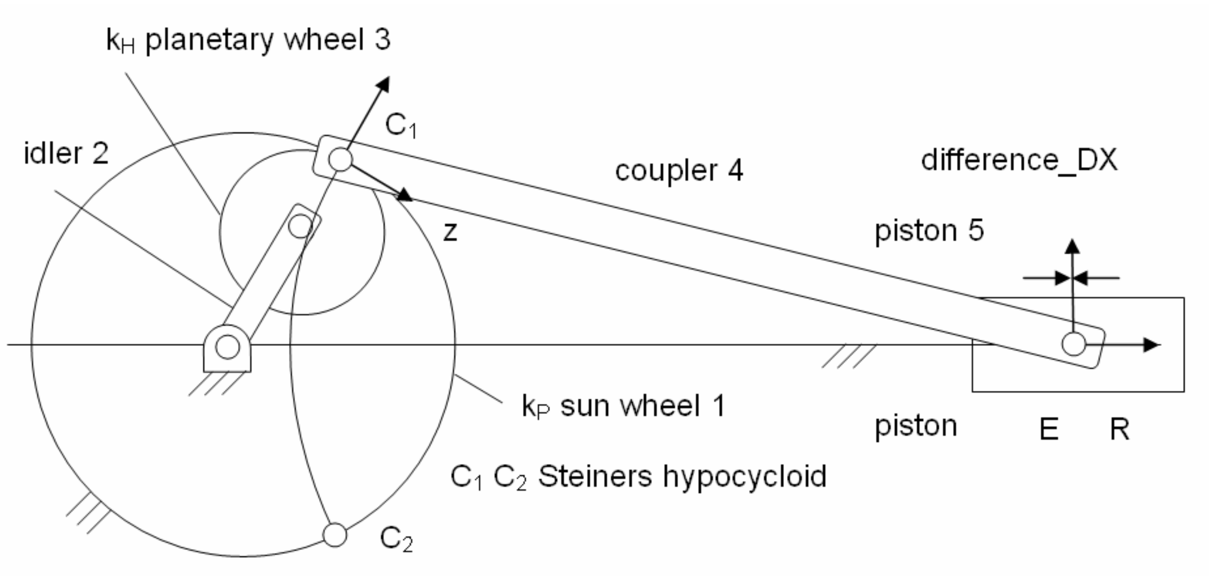


Fig 2. Dwell mechanism with hypocyclic gear train.

**Actual mobility $n_S$**

It is to determine the actual mobility $n_S$ of hypocyclic gear train. Let us first determine mobility $n$ from the formula

$$n = n_v(u-1) - \sum_{t=1}^{t_m} t\, s_t \qquad (1)$$

In our example the mobility $n_v$ of the free body in the plane is $n_v = 3$, total number $u$ of links is $u = 5$.

The number $s_t$ of geometrical constraints (joints) of the class $t$ of all interconnected pairs of links in our dwell mechanism we obtain from the formula

$$s_t = \sum_{v=2}^{v_m} s_{tv}(v-1)$$

where $s_{tv} = s_{22}$ is the number of joints of the type $t = 2$ connecting the number $v = 2$ links.

$$s_2 = s_{22} = 4(R) + 1(P) + 1(V) = 6$$

R - revolute joint, P – prismatic joint, V – rolling joint

Substituting into formula (1) we obtain mobility $n = 0$ which is different as actual mobility $n_S = 1$, because the rolling joint of the type $t = 2$ is partially passive and removes only number $n_O = 1$ of DOF, $n_O = 1 < t = 2$. Our incorrect dwell mechanism has in reality actual mobility $n_S = n + n_N = 1$ because $n_N = 1$ is number of unremoved DOF in the rolling joint.

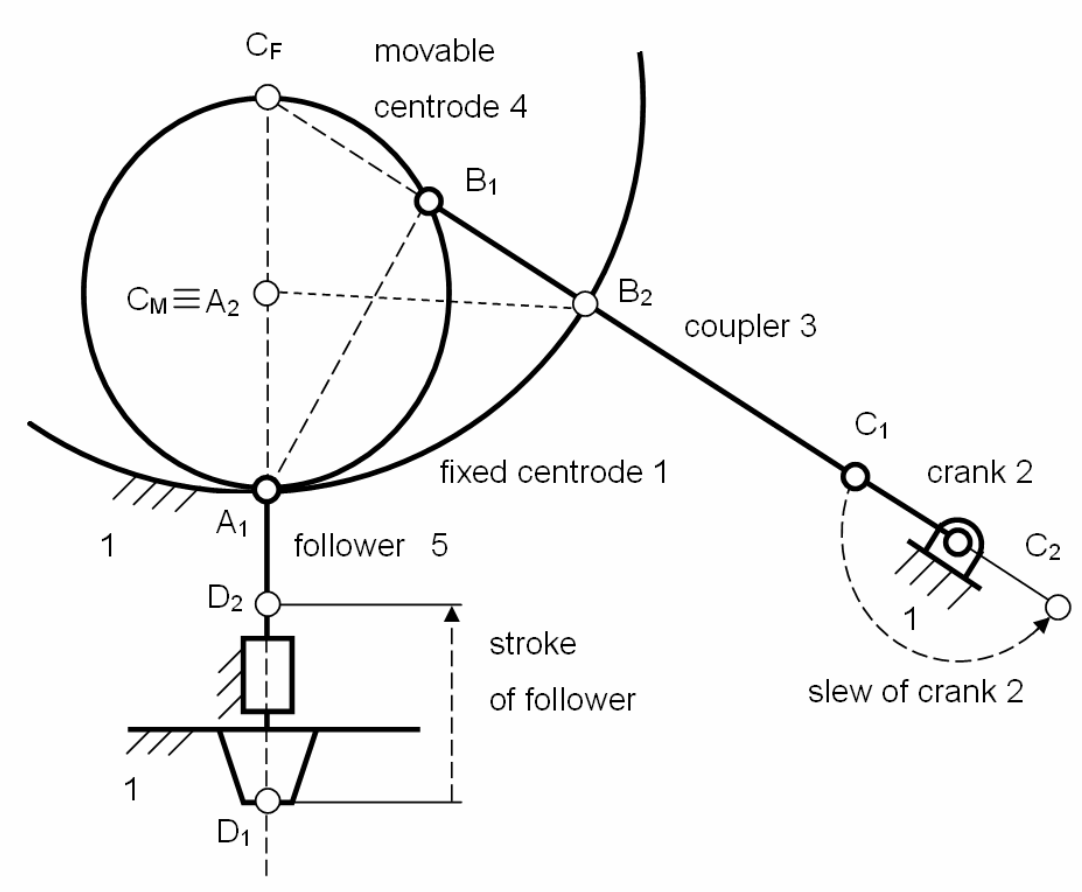


Fig 3. The valve closing mechanism with straight paths of points A and B.

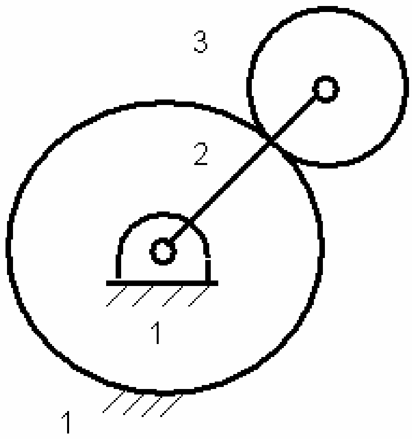


Fig 4. The epicyclic planetary gear train mechanism.