2-5532 Theory of Mechanisms

Applied Mechanics and Mechatronics for bachelor study, year 3, summer sem.
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Lecture 1: Introduction to theory of mechanisms

Sections in Lecture 1:

S1 The goals of the course Theory of Mechanisms
S2 Structural parameters of correct multibody systems (MBS)
S3 Incorrect MBS

S1 The goals of the course Theory of Mechanisms

Lectures of course Theory of Mechanisms brings explanation of theoretical background necessary for kinematic analysis and synthesis of rigid multibody systems. Tutorials are dedicated enable solution of samples in which theory is applied. Art and human insight combined with precise algorithm or recipe is necessary for integration of the applicability of computers for improved comprehension, rapid experimentation and genuine robust optimization of virtual prototypes with real complexity of properties of multibody systems.

Statics	The first part of the study of Engineering Mechanics is devoted to Statics, which is concerned with the equilibrium of bodies at rest or moving with constant velocity.
Dynamics	The second part of the study of Engineering Mechanics is devoted to Dynamics, which is concerned with bodies having accelerated motion. The two aspects of Dynamics are Kinematics and Kinetics. Kinematics treats only the geometric aspects of motion and Kinetics is devoted to the analysis of the forces causing or keeping the motion and to the modal analysis.
Kinematics	Kinematics is concerned only with the motion of bodies with geometric constraints, irrespective of acting forces. In the Kinematics are tasks where only geometric and kinematic properties of constrained particles and rigid bodies are considered, and the kinematics constraints determine what you are interested in, independent of the forces or time history. A classic example is determining the path of a point on a given four-bar linkage with prescribed motion of input link. More basic examples include finding position or acceleration from a given velocity history. For kinematics calculations is necessary to pick appropriate configuration variables, as many as there are degrees of freedom. Then write the velocities, accelerations, angular velocities and angular accelerations of interest in terms of the configuration variables and their first and second time derivatives, possibly using methods from chapter devoted to the mechanics of constrained particles and rigid bodies.

Kinetics

The Kinetics comprises direct and inverse dynamics analysis and also modal analysis. According to desired target there exist analyses in time, frequency and modal domain.
For direct dynamics analysis you are given some information about forces, constraints, inertial properties of components and you have to find the motion of components and more about the constraint forces. The differential equations from the balance laws have to be solved.
In opposite to direct dynamics analysis is “inverse dynamics.” These tasks are called “inverse” because they are backwards of the direct dynamics tasks. The motion is given as a function of time, and you have to calculate the generalized forces required to keep that motion and determine the constraint forces. These tasks are easier than direct dynamic analysis beause the differential equations from the balance laws don’t need to be solved.
In modal analysis, calculation of normal modes requires the use of eigenvalues and eigenvectors.

Engineering Mechanics

In terms of putting all the ideas together these are the capstone tasks that require use of all the skills.

S2 Structural parameters of correct MBS

Position in plane

Cauchy (1827) and Poisson (1834) stated that the general plane motion of the free body can be replaced by the fictive translation represented by arbitrary chosen reference point and by the fictive rotational motion with centre in this reference point.
According to Cauchy-Poisson statement the unique position of free, unconstrained body PART2 on Fig.1 (represented by local coordinate system-LCS) with respect to (wrt) PART1 (represented by global coordinate system-GCS) in plane is given by two Cartesian position coordinates (x_A, y_A) of reference point $A\,(x_A, y_A)$ identical with origin O_2 wrt origin O_1 of GCS and by variable (floating) angle $j_{12} =< (x_1, x_2)$ for angular displacement (slew) of LCS axis x_2 wrt axis x_1 of GCS.

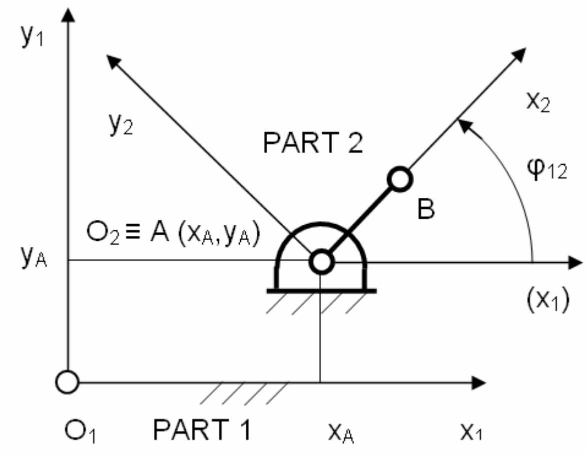


Fig.1 Position of PAR2 wrt PAR1 in plane

Position in space

Cauchy (1827) and Poisson (1834) stated that the general spatial motion of the free body can be replaced by the fictive translation represented by arbitrary chosen reference point with three Cartesian position coordinates (longitudinal displacements x,y,z) and by the fictive spherical motion with centre in this reference point represented by three position angles for sequence of the slews (for example angular displacements by Euler, resp. Cardan angles).

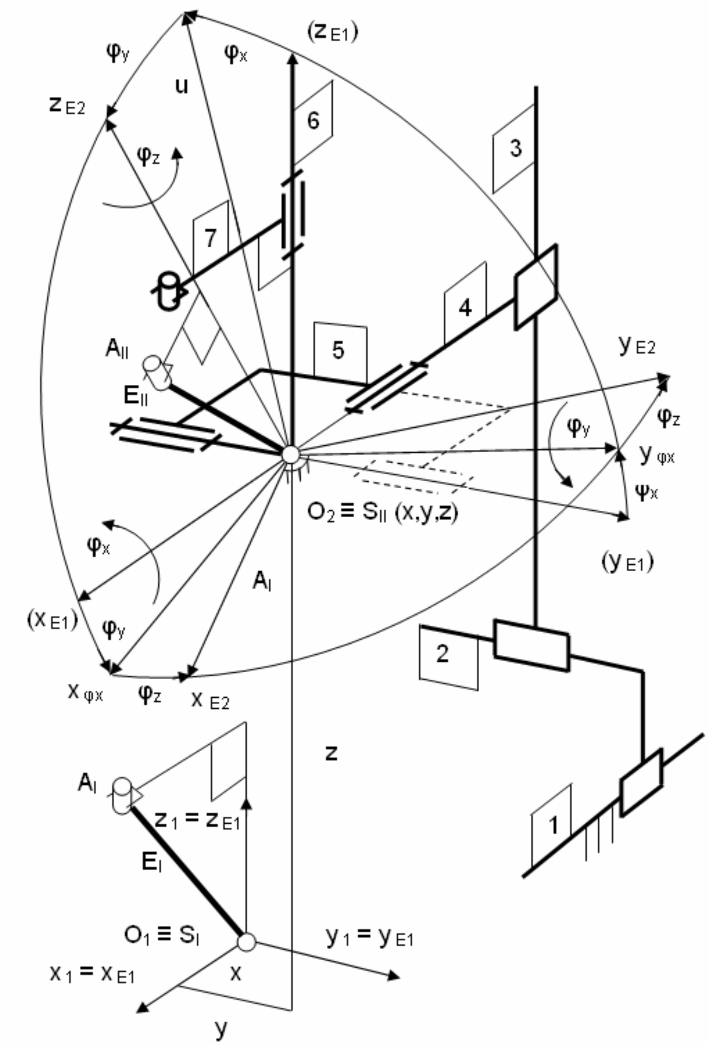


Fig.2 Position of the body $P \equiv E$ frame is determined by three Cartesian position coordinates (x,y,z) of its reference point S and Cardan's angles $(\varphi_x, \varphi_y, \varphi_z)$ derived from given initial $P_I \equiv E_I$ and final $P_{II} \equiv E_{II}$ position of the body $P \equiv E$.

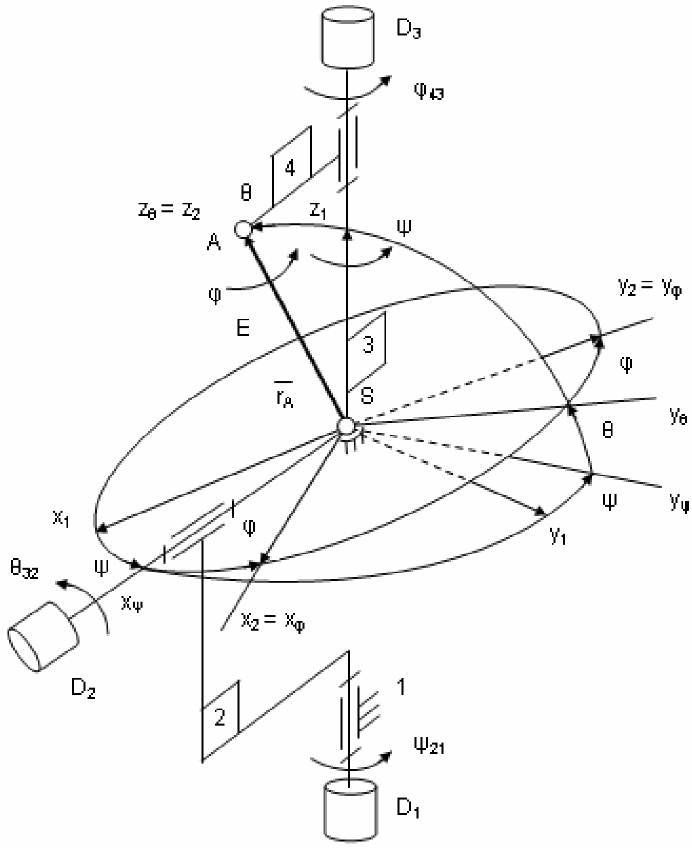


Fig.3 Initial and final position of body E frame determined by Euler's angles y ,q ,j .

Mobility n_b — The number n_b of mutually independent position coordinates is equal to the mobility n_b (or degrees of freedom DOF) of the free particle, so the free particle in the space has mobility $n_b = 3$. The particle constrained to the plane has mobility $n_b = 2$ and constrained to the curve has mobility $n_b = 1$.

Mobility n_v — The number of mutually independent position coordinates is equal to the mobility n_v (or degrees of freedom DOF) of the free body, so the free body has mobility $n_v = 3$ in the plane and mobility $n_v = 6$ in the space.

Single joint — The single joint (holonomic geometrical constraint) is the assembly of pairing elements (surfaces, lines or points) of two links, which are held in contact by means of
- particular geometrical shapes (form closure),
- external forces (force closure),
- flexible materials (material closure).

Multiple joint — Multiple joint is connecting the number of v links, and $v > 2$ with total number p of all possible pairs of links, $p = \frac{v(v\text{-}1)}{2}$, from which number $(v\text{-}1)$ of pairs of links is independent.

Type t of the joint

The type t of the joint (geometrical constraint) is the number t, by which is reduced the mobility n_v of the former free link after its entry into this joint.
The type t of the joint (geometrical constraint) is also equal to the number of contact points, in which surfaces of adjacent links touch in corresponding generalized models introduced by Soni (see literature [2], p.15, or [5], p.2).

Notation of joints

Names (nomenclature) and shortcuts (acronyms) for joints:

- type $t=1$ (in space): general (G), plane-sphere (F_S), plane-peak (F_P), cylinder- cylinder (C_C),
- type $t=2$ (in space): sphere-groove (S_G), sphere-groove-helix (S_{GH}), plane-cylinder (F_C),
- type $t=3$ (in space): spherical (S), sphere-slotted-cylinder (S_{SC}), sphere-slotted-helix (S_{LH}), plane-plane (F_F),
- type $t=4$ (in space): torus-torus (T_T), sphere-slotted (S_L), cylindrical (C), plane-slipping surface (F_K),
- type $t=5$ (in space): revolute (R), prismatic (P), helical (H),

Multibody System

Multibody System MBS (assemblage, or constrained spatial system) is system of bodies (links) whose mutual movement is bounded by geometrical constraints (joints).

Number s_t

Number s_t is the number of geometrical constraints (joints) of the type t of all interconnected pairs of links in the multibody system

$s_t = \sum_{v=2}^{v_m} s_{tv}(v\text{-}1)$, where

s_{tv} is the number of joints of the type t connecting the number v of links

v_m is maximum number of joined links by one geometrical constraint in the multibody system.

Number s

Number of s is the total number of all interconnected pairs of links in the multibody system

$s = \sum_{t=1}^{t_m} s_t$, where

t_m is the maximum type t of the joint (geometrical constraint) in the multibody system

Type g of the link

The type g of the link is the number of adjacent links interconnected by this link. Unary link is of type $g=1$, binary link is of type $g=2$, ternary link is of type $g=3$.

Number u

Number u is the total number of all links in the multibody system

	$u = \sum_{g=1}^{g_m} u_g$, where u_g is the number of links of the type g in the multibody system g_m is the maximum type g of the link in the multibody system.
Local mobility n_t	The local mobility n_t of the link in the joint of the type t is equal to the number of independent position coordinates of link wrt adjacent link in the joint and results from the subtraction $n_t = n_v - t$
Drawing	Construction drawing and production drawing is descriptive model for intended physical construction.
Kinematic diagram	Kinematic diagram (stick diagram, skeleton diagram) displays only essential skeleton of the physical construction of the multibody system.
Sketch	Sketch is more or less proportional kinematic diagram of the multibody system, but not exactly to scale.
Scaled diagram	Scaled diagram (metric model) is proportional kinematic diagram to the drawing, or physical construction of the multibody system.
Structural diagram	Structural diagram is scheme of structure (topological model) of the multibody system which does not contain metric data about dimensions and mutual configuration of links in space.
Unicomponential MBS	Each link in the unicomponential multibody system (assemblage) is connected to its neighboring link by the geometrical constraint (joint).
Kinematic chain	Kinematic chain (KR) of the links is any unicomponential multibody system without frame.
Closed chain	Closed chain (UR) of the links has property $s = u$ with symbolic description of its structure by numbers 1234 (for example).
Kinematic loop	Kinematic loop (KS) of the links differs with closed chain (UR) only in the symbolic description of its structure by numbers 12341 (for example), where the first number is redoubled at the end of description.
Number k_s	Number k_s is number of all possible kinematic loops (KS) in the multibody system.

Open chain OR	Open chain (OR) has property $s = u - 1$
Open chain OROV	Open chain with joint elements at both tails (OROV) has property $s = u + 1$.
Single-loop chain	Single-loop chain (JR) has $k_s = 1$.
Multi-loop chain	Multi-loop chain (VR) has $k_s > 1$.
Combined chain	Combined chain (KR) has minimum one UR and one OR.
Basic chains	Basic chains ZJR and ZOROV are developed by decomposition of multi-loop chain (VR) into one ZJR and the number $k-1$ ZOROV, where k is the number of the basic kinematic loops ZKS.
Number k	The number k of the basic kinematic loops ZKS is according to the Euler invariant property related to the number s and u: $k = s - u + 1$
Mobility n	Under the term mobility n of multibody system we mean the number of prescribed independent position coordinates of input (driving) links required to uniquely determine dependent position coordinates of driven output links $n = n_v(u-1) - \sum_{t=1}^{t_m} t\, s_t$
Multibody system	When one link in the kinematic chain is fixed (it becomes the frame) then arose the multibody system (assemblage).
Structure	Structure is multibody system (assemblage) with mobility $n \le 0$. If mobility of structure is $n = 0$, the structure is statically determinate, and in the case, when mobility of structure is $n < 0$, the structure is statically indeterminate.
Mechanism	Mechanism is multibody system (assemblage) with mobility $n \ge 1$. It is a mechanical device that has the purpose of transferring motion and/or force from a source (single input link if $n = 1$, or more input links if $n \ge 1$) to an output (single output link, or more links).
Types of mechanisms	According to type of structure of corresponding kinematic chain we analogically denote closed mechanism (UM), single-loop mechanism (JM), multi-loop mechanism (VM), open mechanism (OM) and combined mechanism (KM).
Linkage	Linkage is a mechanism which all joints are of the type $t = 5$ (space mechanisms), or $t = 2$ (planar mechanisms).

Cognate mechanisms — Geometrically different mechanisms are cognate, when they have the same transfer function.

Isomorphic diagrams — If different mechanisms have equal structural diagrams, these diagrams are isomorphic.

Coordinate systems — Each link (part) has local (own, or intrinsic) orthonormal reference coordinate system.

Local coordinates — By the local position coordinates of the link (part) is described mutual local relative position of link wrt adjacent (neighboring) link in the geometric constraint (joint). Local position coordinates can be in the form of

- variable (floating) abscissa $\overline{q}_{ij} = \overrightarrow{O_iO_j}$ for longitudinal displacement of part reference frames, and
- variable (floating) angle $\overline{\varphi}_{ij} = \sphericalangle(x_i, x_j)$ for angular displacement of part reference frames.

Global coordinates — By the global position coordinates of the links in assemblages is described global relative position of links wrt frame (default is the part 1). Global position coordinates can be in the form of

- variable (floating) abscissa $\overline{p}_{1j} = \overrightarrow{O_1O_j}$ for longitudinal displacement of part reference frames, and
- variable (floating) angle $\overline{\psi}_{1j} = \sphericalangle(x_1, x_j)$ for angular displacement of part reference frames.

Number c — Number c is total number of local position coordinates $q_i, i = 1, 2, ..., c$ of the links in the mechanism

$$c = \sum_{t=1}^{t_m} n_t s_t$$

and it is a sum $c = n + z$, where n is number of independent local position coordinates of the links (also n is mobility of mechanism) $q_{n\,i},\ i = 1, 2, ..., n$, and
z is number of dependent local position coordinates of the links $q_{z\,i},\ i = 1, 2, ..., z$.

Number z — Number z is number of dependent local position coordinates of the links

$$z = n_v k$$

Number m — Number m is total number of global position coordinates of the links

$$\psi_i,\ i = 1, 2, ..., m$$

and it is a sum $m = n + d$, where n is number of independent global position coordinates of the links

$y_{n\,i}$, $i = 1, 2, ..., n$,
where n is mobility of mechanism and d is number of dependent global position coordinates of the links
$y_{z\,i}$, $i = 1, 2, ..., d$.

Number d — Number d of dependent global position coordinates of the links result from equation
$d = 2k + s_1$

Relation m and c — There is relation between m and c
$m = c - k + s_1$

Actual mobility n_S — If a multibody system (MBS) has in reality actual mobility n_S which is different as theoretical mobility n computed from formula $n = n_v(u-1) - \sum_{t-1}^{t_m} t\, s_t$, so $n_S \neq n$, then such MBS is called incorrect.

Correct MBS — A multibody system (MBS) with actual mobility $n_S = n$ equal to the theoretical mobility n computed from formula $n = n_v(u-1) - \sum_{t-1}^{t_m} t\, s_t$ is called correct MBS. In a correct MBS each geometrical constraint of type t removes just the same number t DOF.

Example 1 Structural parameters of a given multibody system

Given: Let us consider given double piston-crank mechanism of a engine on Fig.4.

Task: It is to:

1. develop a structural diagram (structural scheme as topological model) of the given double piston-crank mechanism of a engine from Fig.4,
2. determine structural parameters: mobility n of given multibody system, number k of the basic kinematic loops, number d of dependet global position coordinates of the links and number z of dependet local position coordinates of the links.

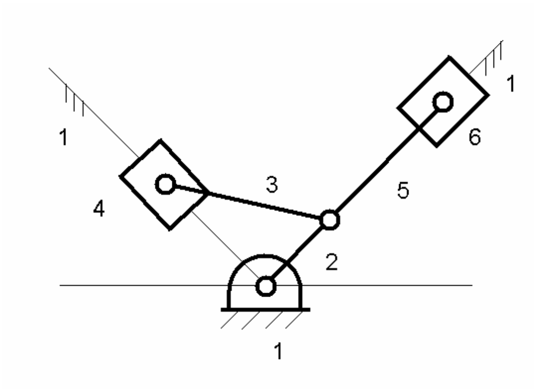


Fig. 4 Double piston-crank mechanism of a combustion engine.

Solution

Solution 1: The scheme of structure on Fig.5 has two windows (loops): k_1 (1,4,3,2) and k_2 (1,2,5,6)

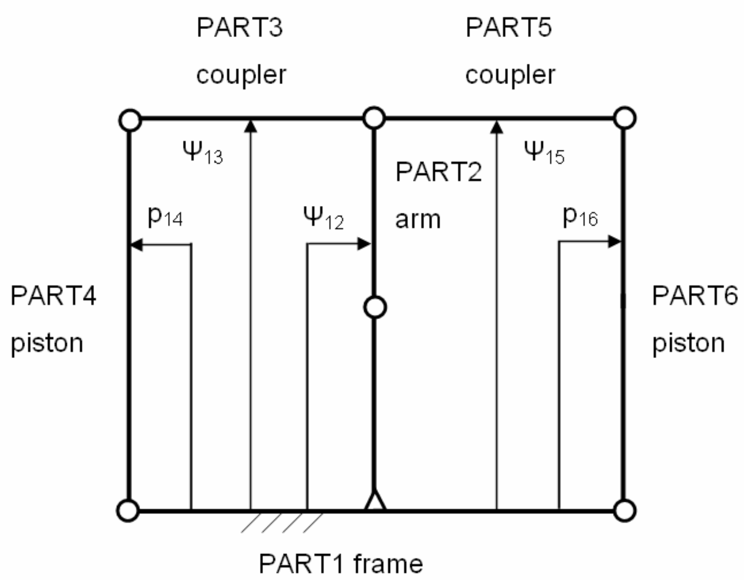


Fig.5 The scheme of structure of double piston-crank mechanism with global position coordinates of the links.

Solution 2: For determination of structural parameters are used formulas from Lecture 1.

The mobility n of given multibody system

$$n = n_v(u-1) - \sum_{t=1}^{t_m} t\,s_t$$

the mobility n_v (or degrees of freedom DOF) of the free body in the plane, $n_v = 3$, number of links $u = 6$,

the type t of the joint (geometrical constraint), $t = 2$ for rotational and prismatic joint,

$t_m = 2$. $s = \sum_{t=1}^{t_m} s_t$, where $s_t = \sum_{v=2}^{v_m} s_{tv}(v-1)$

$v_m = 3$ (joint connecting links 2,3,5)

$s_{22} = 5$, $s_{23} = 1$, so $s_2 = s_{22}(2-1)+s_{23}(3-1)$, then $s = s_2 = 7$

$n = 3(6-1) - 2.7$, $n = 1$.

Answer 1

In the given mechanism is one input in the form of prescribed motion for the piston 4.

Number d of dependet global position coordinates of the links result from equation $d = 2k + s_1$, where $k = s - u + 1$, so $k = 7 - 6 + 1 = 2$, then $d = 2.2 + 0 = 4$ and total number m of global position coordinates is $m = n + d$, so $m = 1 + 4 = 5$ (see Fig.5).

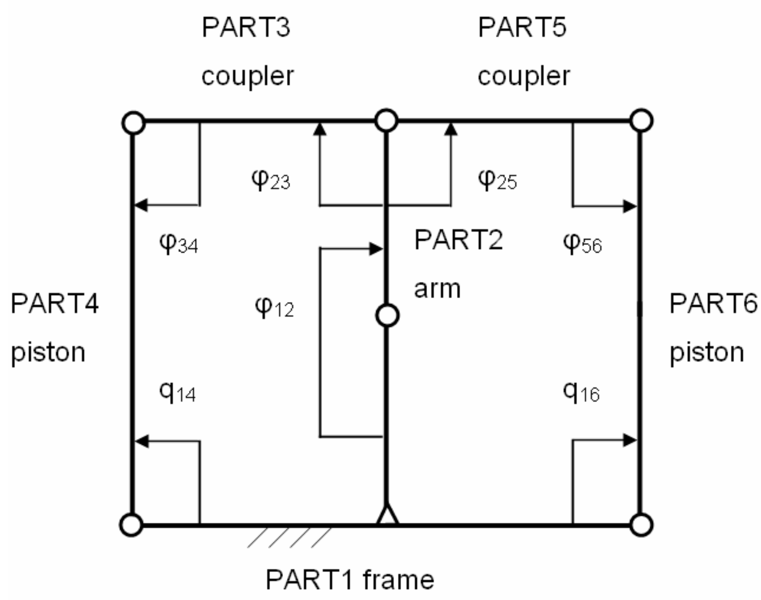


Fig.6 The scheme of structure of double piston-crank mechanism with local position coordinates of the links.

Answer 2 — Number z of dependet local position coordinates of the links result from equation $z = n_v k$, where $k = 2$, then $z = 3.2 = 6$ and total number c of global position coordinates is $c = n + z$, so $c = 1 + 6 = 7$ (see Fig.6).

S3 Incorrect MBS — Incorrect MBS has in reality actual mobility $n_S \neq n$ different as theoretical mobility n. The reason consists in fact, that formula $n = n_v(u\text{-}1)\text{-}\sum_{t\text{-}1}^{t_m} t\, s_t$ does not contain information neither about proportions (metrics) nor about mutual position (configurations) of links and geometrical joints. Theoretical mobility n of incorrect MBS may be zero (indicating a structure) or negative (indicating an indeterminate structure) but it can in reality, nevertheless, move, so its actual mobility $n_s \geq 1$ due to special proportions (metrics) and mutual position (configurations) of links and geometrical joints.

Unremoved DOF — Incorrect MBS has in reality actual mobility $n_S = n + n_N$ where n_N is number of unremoved DOF due to special proportions (metrics) and mutual position (configurations) of links and geometrical joints.

Singularities — Under common term singularities in MBS we denote all reasons (passivity, redundancy, general constraint,

	irregularity,) which causes that $n_S \neq n$, hence actual mobility n_S is different as theoretical mobility n.
Total passivity	A constraint is totally passive if it can be removed and actual mobility of MBS does not change.
Partial passivity	A constraint of a class t is partially passive if it removes from MBS only number n_O of DOF, $n_O < t$.
Over constrained MBS	A MBS with theoretical mobility $n \leq 0$ and actual mobility $n_s \geq 1$ is over constrained when actual mobility does not change after removing totally passive constraint.
Locked MBS	If redundant constraint in MBS become inconsistent with other constraints (due manufacturing differences in link lengths or pivot locations), this causes that MBS will jam (locked).
Local mobility n_L	Local mobility n_L is a passive (redundant) kinematic input which has no influence on the mobility of output link.
Active mobility n_A	Active mobility n_A is a active kinematic input which has influence on the mobility of output link $n_A = n_S - n_L$.
Singular state	A MBS is in instantaneous singular state, when its links can displace with infinitely small values of position coordinates maintaining geometrical constraints. If MBS is at permanent singular state, its links can displace with finite values of position coordinates (gross displacement).

Development of vectorial and scalar loop constraint equations

Goal of kin. analysis	Goal of kinematic analysis of the given planar mechanism with known initial values of all position coordinates of links and prescribed motion of input link/or links is to determine the time course for number $d = 2k + s_1$ of dependet global position coordinates, velocities and accelerations of output links.
Loop equations	The mathematical model with desired number d of lineary independet equations enabling to obtain number $d = 2k + s_1$ of dependet global position coordinates is necessary to develop according to type of mechanism.
Type R, P	For a planar closed mechanism (linkage) with revolute R and prismatic P joints, the number $d = 2k$, ($s_1 = 0$), (k is number of the basic kinematic loops in the structure of given planar mechanism) of unknown dependet global position

coordinates can be determined from number $\mathrm{d} = 2\mathrm{k}$ of explicit scalar loop constraint equations as projections of number k of vectorial loop equations into axes x_1, y_1

$$\mathrm{k}_j \approx \sum_{i=1}^{p_h} \overline{\mathrm{h}}_i = \overline{0}\,,\ \ j = 1,2,\ldots,k\,,\ \ i = 1,2,\ldots,p_h$$

where p_h is the number of oriented edges $\overline{\mathrm{h}}_\mathrm{i}$ in the polygon related to given planar mechanism.

Type: correct K

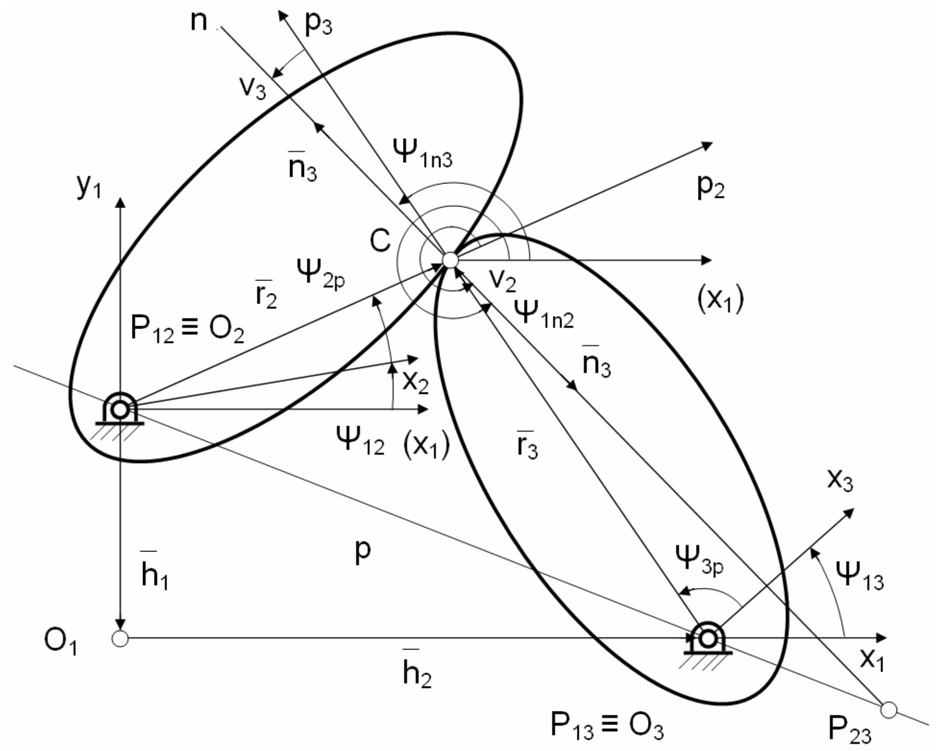
In a planar closed mechanism with correct slipping joint K (geometrical constraint) of class $\mathrm{t} = 1$ in which are mating adjacent links j and $\mathrm{j}+1$, and touch point C is out of pole line, it is necessary add to the number $\mathrm{d} = 2\mathrm{k}$ of explicit scalar loop constraint equations one auxilliary explicit constraint equation

$$y_{1\mathrm{n}(\mathrm{j}+1)} - y_{1\mathrm{n}\,\mathrm{j}} = \mathrm{b}p$$

where $y_{1\mathrm{n}\,\mathrm{j}} = y_{1\mathrm{j}} + y_{\mathrm{jp}} + n_{\mathrm{j}}$, and $y_{1\mathrm{j}} = \sphericalangle(\mathrm{x}_1, \mathrm{x}_\mathrm{j})$ are global position coordinates, and angle $y_{\mathrm{jp}} = \sphericalangle(\mathrm{x}_\mathrm{j}, \mathrm{p}_\mathrm{j})$, where p_j are lines of action for given radius vectors $\overline{\mathrm{r}}_\mathrm{j} = \mathrm{F}_\mathrm{j}(y_{\mathrm{jp}})$, which defines the shapes of contact surfaces, and $n_\mathrm{j} = \sphericalangle(\overline{\mathrm{r}}_\mathrm{j}, \overline{\mathrm{n}}_\mathrm{j})$ are angles between radius vectors $\overline{\mathrm{r}}_\mathrm{j}$ and outward normal vectors $\overline{\mathrm{n}}_\mathrm{j}$.

Angles n_j are determined by the shapes of contact surfaces from equation $n_\mathrm{j} = \mathrm{arctg}_2(\mathrm{y}/\mathrm{x})$, according to signum of projections y and x in the unit circle.

The coefficient b is resulting from initial mutual position of mating contact surfaces.



Obr.7 Correct slipping joint K (geometrical constraint) of class $t = 1$ in cam mechanism.

Type: incorrect K

If in a planar closed mechanism occurs incorrect slipping joint K (geometrical constraint) of class $t = 1$ in which the touch point C of mating adjacent circular shapes of links j and $j+1$, is out of pole line, then this permanent singular configuration causes total passivity of this slipping joint. Actual mobility n_S of such planar closed mechanism with number r_K incorrect slipping joints is

$n_S = n + r_K \; n_N$, where n is mobility of correct mechanism

Type: closed V

In a planar closed mechanism the closed rolling joint V is of class $t = 2$, but singular configuration of adjacent links j and $j+1$ with mating circular shapes causes that rolling joint is incorrect, partially passive, with number $n_N = 1$ of uneliminated degrees of freedom. So actual mobility n_S of mechanism with number r_V of closed rolling joints is then

$$n_S = n + r_V \; n_N$$

where n is mobility of mechanism evaluated under assumption of its correctness.

Type: open V

In a planar closed mechanism with open rolling joint V each open rolling joint V should be transformed into closed rolling joint imposing the auxilliary fictive binary link into mechanism.

Auxilliary equation

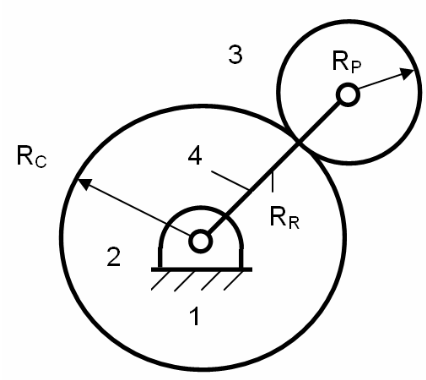
Each basic loop in mechanism with closed rolling joints of links with circular shapes degenerates into abscissa. Because the number $d = 2k + s_1$ of dependet global position coordinates have to be determined, it is necessary add to the number $d = 2k$ of explicit scalar loop constraint equations one auxilliary explicit constraint equation of pure rolling condition resulting from basic equation of planetary (epicyclic) gear train

$R_C w_{1C} = (R_P + R_C) w_{1R} - R_P w_{1P}$, where

R_C is diameter of sun gear with angular velocity w_{1C},

R_R is length of arm (spider, or carrier) rotating with angular velocity w_{1R},

R_P is diameter of planet gear with angular velocity w_{1P}.



Obr.8 Planetary (epicyclic) gear train mechanism.

Mobility of MBS

Mobility of MBS can be determined from point of view of Statics, Kinematics and Dynamics.

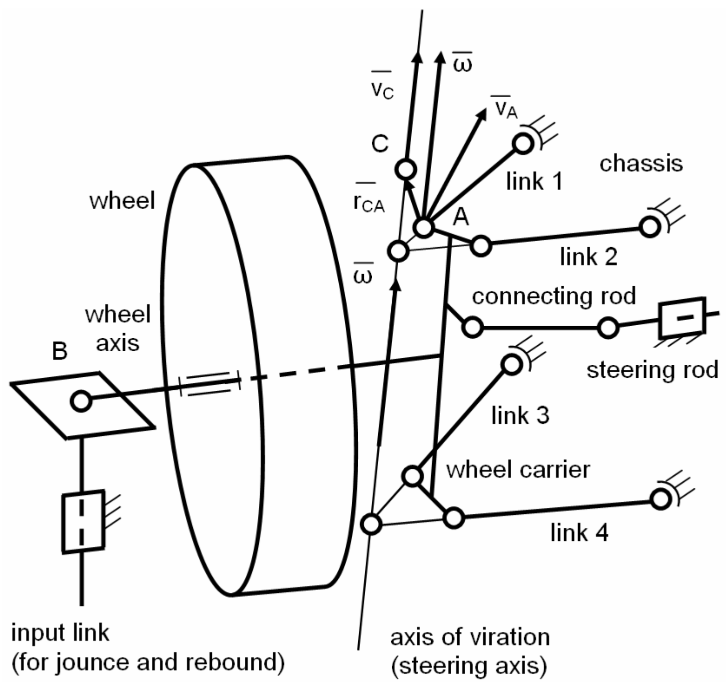


Fig.9 Spatial mechanism of five-links front wheel suspension.