2-5520 Theory of Mechanisms

Glossary

for bachelors study in 3rd year-classis, summer semester

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Lecture 10: General spatial motion of a body.

Sections in lecture 10:

S1 Poissont‘s decomposition of general spatial motion of a body to the fictive translation of a body represented by reference point and to the fictive spherical motion of a body about reference point.

S2 Chasle‘s decomposition of general spatial motion of a body by instantaneous tangential screw motion of a body wrt axis of viration.

S3 Applications of general spatial motion of a body in mechanisms.

S1 Poissont‘s decomposition of general spatial motion of a body

Spatial motion — The position of the free (unconstrained) body $P \equiv E$ in the space wrt reference part ground, can be specified by the six mutually independent position coordinates $(x,y,z),(\varphi_x,\varphi_y,\varphi_z)$ because the number of mutually independent position coordinates is equal to the mobility $n_v = 6$ (or degrees of freedom DOF) of the free (unconstrained) body.

Poissont‘s method — The open mechanism on Fig.1 consisting from number $u = 7$ of links, number $s_{22} = 3(T) + 3(R)$ of geometrical constraint and with mobility $n = 6$ is used for demonstration of Poissont‘s decomposition of general spatial motion of a body $P \equiv E$ from given initial $P_I \equiv E_I$ position $S_I(x_{E1},y_{E1},z_{E1})$ to the final $P_{II} \equiv E_{II}$ position $S_{II}(x_{E2},y_{E2},z_{E2})$.

For fictive translation is body $P \equiv E$ represented by it´s reference point S (the origin of the local coordination system (LCS) of the body $P \equiv E$). During fictive translation of the body $P \equiv E$ from given initial $P_I \equiv E_I$ to the final $P_{II} \equiv E_{II}$ position the reference point S is dispalced stepwise along axes x_1,y_1,z_1 by the three longitudinal displacements x,y,z resulting from coresponding Carthesian position coordinates $S_{II}(x,y,z)$.

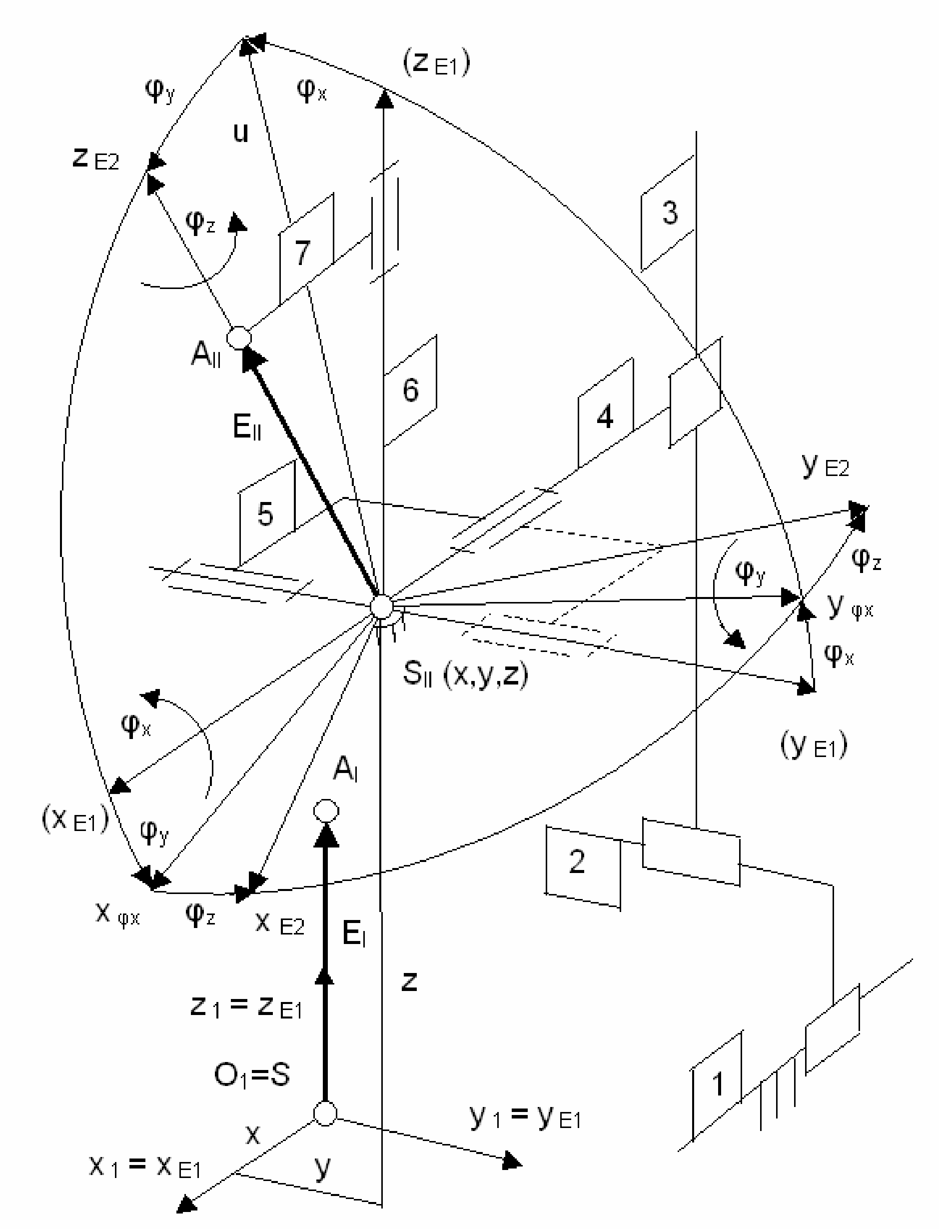

Fig.1 Position of the body $P \equiv E$ frame is determined by three Carthesian position coordinates (x,y,z) of it´s reference point S and Cardan´s angles $(\varphi_x, \varphi_y, \varphi_z)$ derived from given initial $P_I \equiv E_I$ and final $P_{II} \equiv E_{II}$ position of the body $P \equiv E$.

Cardan´s angles

The fictive spherical displacement of a body $P \equiv E$ about reference point S_{II} is realized as a sequence of three slews by three Cardan´s angles $(\varphi_x, \varphi_y, \varphi_z)$. On the Fig.1 the position of the line $u \equiv (y_{E1}, z_{E1}) \times (x_{E1}, z_{E2})$ of intersection of both frame planes provide the first angle $\varphi_x \equiv (z_{E1}, u)$, the second angle is $\varphi_y \equiv (u, z_{E2})$, and last one is $\varphi_z \equiv (x_{E1}, \varphi_y)$. There are so-called 1-

2-3 Cardan´s angles $(\varphi_x, \varphi_y, \varphi_z)$, advantageous for description of small values of angles.

Euler´s angles

It is possible uniquely describe given final position $(O_2,x_2,y_2,z_2)_{II}$ of the body E after spherical displacement of a body $P \equiv E$ about reference point S_{II} from given initial $P_I \equiv E_I$ position $S_I(x_{E1},y_{E1},z_{E1})$ to the final $P_{II} \equiv E_{II}$ position $S_{II}(x_{E2},y_{E2},z_{E2})$ as a sequence of three slews by three independent position coordinates (so-called 3-1-3 Euler´s angles $y\ ,q\ ,j$). The line $x_\psi \equiv (x_1,y_1) \times (x_2,y_2)$ of intersection of both frame planes provides us by angle $y \equiv (x_1, x_\psi)$, or $y \equiv (y_1, y_\psi)$, which should be applied as a first slew (precession) of local frame of body E about axis z_1. The angle $q \equiv (z_1, z_2)$ which yield from mutual position of axes z_1, z_2 or $q \equiv (y_\psi, y_\theta)$ is applied for a second slew (nutation) of local frame of body E about axis x_ψ. The local frame of body E will achieve it´s final position $(O_2,x_2,y_2,z_2)_{II}$ after application of third slew (spin) about axis z_2 by angle $y \equiv (x_\psi,x_2)$, or $y \equiv (y_\theta,y_2)$.

S2 Chasle‘s decomposition of general spatial motion of a body by instantaneous tangential screw motion of a body wrt axis of viration.

Chasle‘s method

On the Fig.2 is depicted the Chasle‘s decomposition of general spatial motion of a body P represented by radius vector $\overrightarrow{AL}$ into fictive translation of a body P along instantaneous axis o_ω of viration and into fictive instantaneous rotation of a body P about instantaneous axis o_ω of viration. The general spatial motion of a body P is then represented by the instantaneous tangential screw motion wrt axis of viration o_ω characterized by couple $\overline{\omega}$ and $\overline{v}_C$ of collinear vectors.

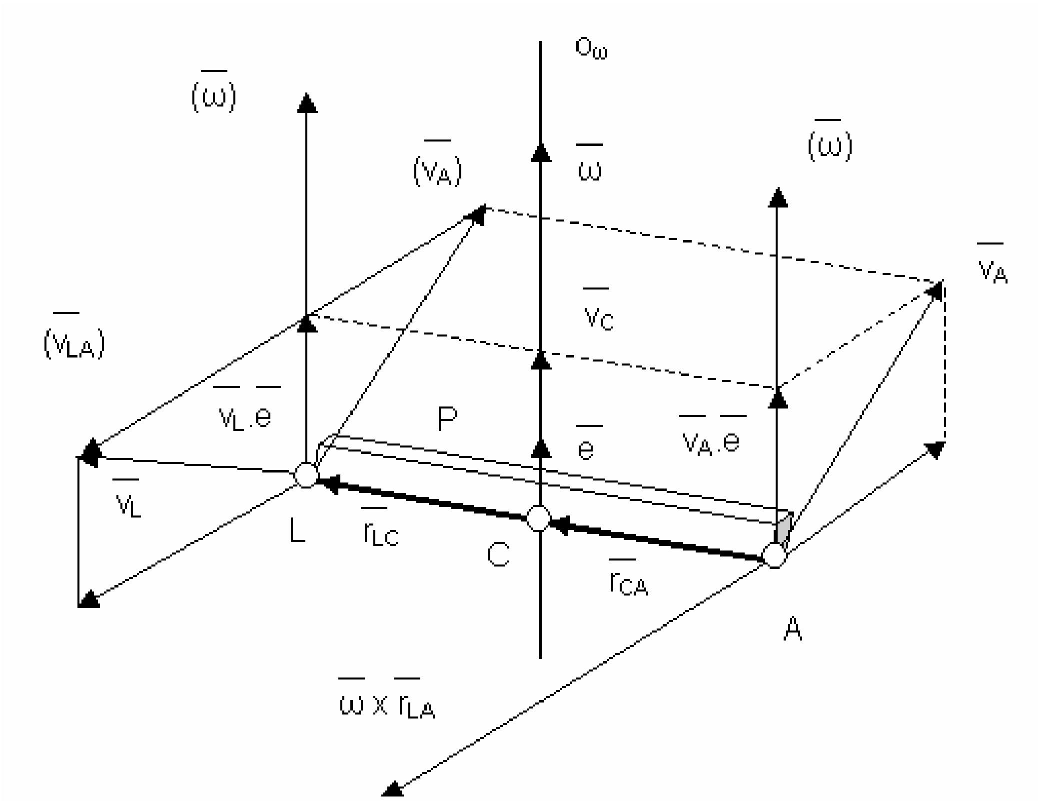

Fig.2 Tangential screw motion of a body P about the axis of viration o_ω , line of action of the couple $\overline{\omega}$, $\overline{v}_C$ of collinear vectors.

Screw motion

Applying the method of Poissont‘s decomposition on general motion of a body P with points A and L, the instantaneous velocity $\overline{v}_L$ of the point L can be composed from instantaneous velocity $\overline{v}_A$ of reference point A, representing translation of a body P and from velocity $\overline{v}_{LA} = \overline{\omega} \times \overline{r}_{LA}$ of rotating radius vector $\overline{r}_{LA}$ in the form

$$\overline{v}_L = \overline{v}_A + \overline{\omega} \times \overline{r}_{LA} \qquad (1)$$

or

$$\overline{v}_L = \overline{v}_A + \overline{v}_{LA}$$

After dot product of Eq.1 by unit vector $\overline{e}$ we obtain the generalized Kováč's invariant property for rotational field of velocities of body P points

$$(\overline{v}_L \, g \, \overline{e})\overline{e} = (\overline{v}_A \, g \, \overline{e})\overline{e} = \overline{const} = \overline{v}_{min} = \overline{v}_C \qquad (2)$$

Now it is to find the position of point C from body P for which is valid the condition of collinearity:

$$\overline{\omega} \times \overline{v}_C = \overline{0} \qquad (3)$$

Again from the Poissont‘s decomposition for point C we obtain

$$\overline{v}_C = \overline{v}_A + \overline{\omega} \times \overline{r}_{CA} \tag{4}$$

Aplying cross product on Eq.4 by $\overline{\omega}\times$ from left yield

$$\overline{\omega} \times \overline{v}_C = \overline{\omega} \times \overline{v}_A + \overline{\omega} \times (\overline{\omega} \times \overline{r}_{CA}) \tag{5}$$

Considering the condition (3) we obtain the position vector of the point C wrt point A:

$$\overline{r}_{CA} = \frac{\overline{\omega} \times \overline{v}_A}{\omega^2} \tag{6}$$

The point C lays on the axis of viration o_ω, line of action for couple $\overline{\omega}$ and $\overline{v}_C$ of collinear vectors. Instantaneous velocity $\overline{v}_C$ of translation of body P and angular velocity $\overline{\omega}$ of rotation of body P about axis of viration are related by slope k_O of screw trajectories of all points of body P out of axis of viration, so $\overline{v}_C = k_0 \overline{\omega}$ which characterize tangential screw motion of a body P.

S3 Applications of general spatial motion of a body in mechanisms.

Axoids

Axoids on Fig.3 are locus of axes o_ω of a viration in space of a moving body 3, respectively of a reference body 2. Axis of viration o_ω is line of action of instantaneous angular velocity $\overline{w}$ which is tangent of a movable axoidal hypoid h_3 performing screw motion against reference axoidal hypoid h_2 during spatial motion of hypoid h_3 wrt reference hypoid h_2.

Applications

The hypoid pump on Fig.3 is an application of Chasle‘s decomposition of general spatial motion of a body in mechanisms into the instantaneous tangential screw motion.

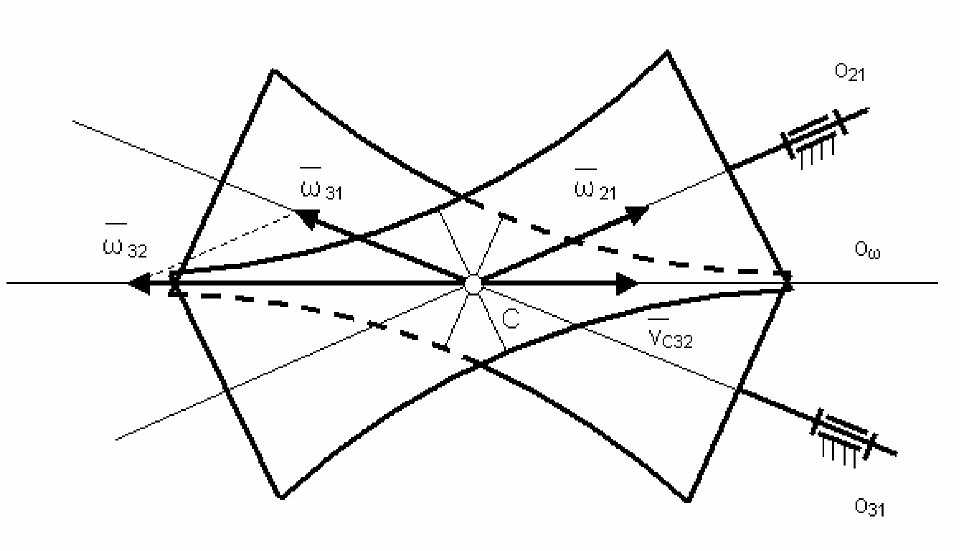

Fig.3 Axis of viration o_ω as tangent of axoidal hypoid h_3 performing screw motion against reference axoidal hypoid h_2

The hypoid pump

The principle of hypoid pump from Fig.3 is mutual tangential screw motion of a movable axoidal hypoid h_3 rotating by angular velocity $\overline{w}_{31}$ and performing screw motion 3/2 against reference axoidal hypoid h_2 rotating by angular velocity $\overline{w}_{21}$.

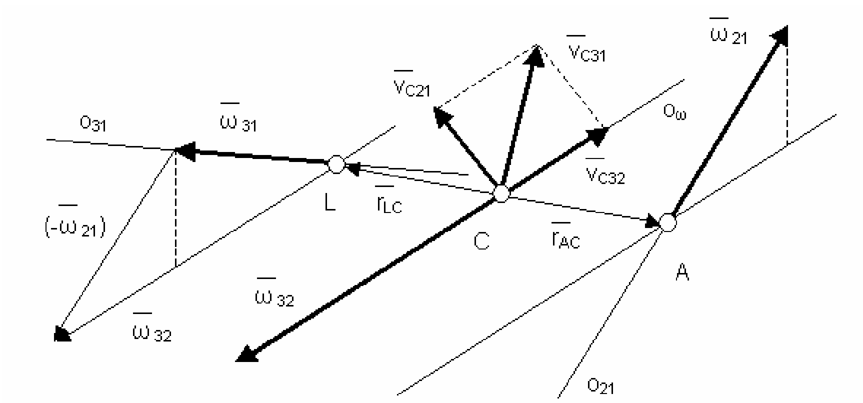

Fig.4 Axes o_{31} and o_{21} are skew lines with transversal $\overrightarrow{AL}$. The couple $\overline{w}_{32}$ and $\overline{v}_{C32}$ of collinear vectors lay on the axis of viration o_ω.

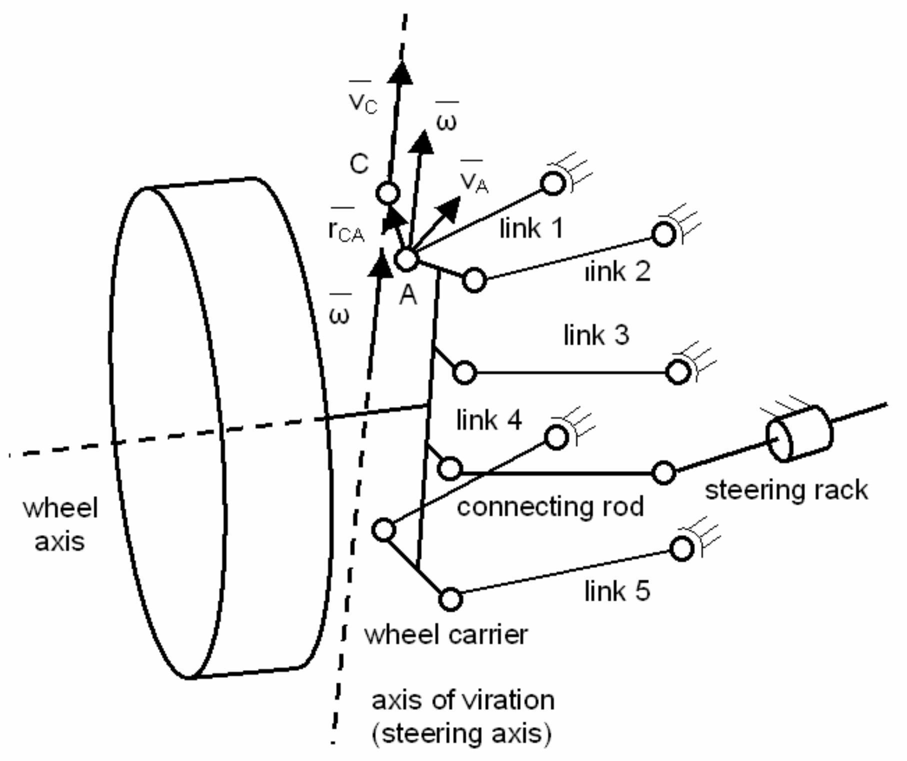

Fig.5 Instantaneous tangential screw motion of a wheel carrier of the front suspension with axis of viration o_ω as steering axis

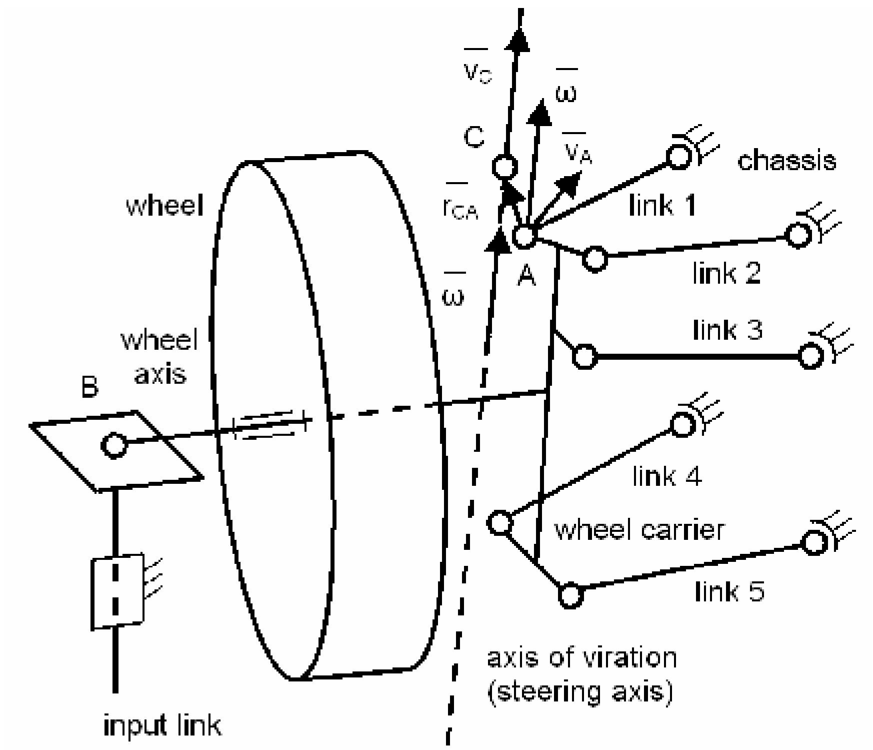

Fig.6 Instantaneous screw motion of a wheel carrier of the rear suspension with axis of viration o_ω as steering axis. Points A, B, C belongs to the wheel carrier.